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Methods for Flutter Stability Analysis of Long-span Bridges: A Review

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Predicting the phenomenon of aerodynamic instability is essential for the analysis and design of long-span cable-supported bridges. This paper reviews the history and development of aerodynamic analysis techniques and state-of-the-art numerical and experimental methods for flutter stability analysis. Discussion on flutter phenomenon is commenced followed by the literature review. This study provides a perspective on the self-excited aerodynamic force models, identification of aerodynamic derivatives and methods for determining aerodynamic instability of long-span bridges in 2D and 3D. Computational Fluid Dynamics (CFD) techniques for modelling the flow past bluff bodies are also covered. Different model combination techniques are presented utilising analytical, numerical and experimental approaches to predict the flutter limit. The effect of different parameters on the flutter stability limit is also highlighted. Furthermore, an overview of the complementary relationship between wind tunnel testing and CFD is provided. Finally, this paper also describes the engineering solutions adopted as countermeasures to prevent flutter instabilities.

14 Notations

St :	Strouhal number
b :	half chord
B :	chord
C_D, C_L, C_M :	static wind coefficient for drag, lift and moment
C'_D, C'_L, C'_M :	derivative of static wind coefficient for drag, lift and moment w.r.t. wind angle of attack
F_D, F_L, F_M :	drag force, lift, force and moment
$h, \dot{h}, \ddot{h}$:	vertical displacement, velocity and acceleration
$\alpha, \dot{\alpha}, \ddot{\alpha}$:	rotational displacement, velocity and acceleration
p :	lateral displacement
ρ :	mass density of the air
U_∞ :	oncoming wind velocity
k, K :	reduced frequency
C :	Theodorsen's circulation function
F :	real part of Theodorsen's circulation function
G :	imaginary part of Theodorsen's circulation function
ϕ :	Wagner indicial function
τ :	reduced time
m, I :	mass and mass moment of inertia
f :	oscillation frequency
ω :	natural circular frequency of oscillation
ω_h, ω_α :	natural circular frequency for heave and pitch
ξ_h, ξ_α :	damping ratio for heave and pitch
$\ddot{x}, \dot{x}, x$:	acceleration, velocity and displacement
H_i^*, A_i^*, P_i^* :	Scanlan's derivatives (with $i = 1..6$)
$k_i^a, k_i^b, m_i^a, m_i^b$:	Küssner coefficients (with $i = 1, 2$)
$\mathbf{M}, \mathbf{C}, \mathbf{K}$:	modal mass, modal damping and modal stiffness
$\mathbf{C}_{ae}, \mathbf{K}_{ae}$:	aerodynamic damping and aerodynamic stiffness
α_e :	effective angle of attack
U_r :	relative velocity
ν :	kinematic viscosity
Δt :	time step
$I_{Q,\alpha}, I_{Q,h}, I_{Q,p}$:	aerodynamic impulse response functions (with $Q = L, M, D$)
$\Phi_{Q,\alpha}, \Phi_{Q,h}, \Phi_{Q,p}$:	aerodynamic indicial functions (with $Q = L, M, D$)
h_o, α_o :	vertical displacement and rotational displacements for forced oscillation simulations
U_d :	divergence velocity
θ :	wind angles of attack
K_α :	torsional stiffness of the deck
r :	radius of gyration
η :	empirical form factor

1. Introduction

The aeroelastic behaviour of long-span cable-supported bridges is very important to be studied as part of the analysis and design. The increased flexibility makes these bridges highly sensitive to the wind action, hence aerodynamic performance often becomes a decisive factor in the design process. Several aspects are highlighted in this paper including a brief history of long-span bridges, description of aeroelastic instability phenomena, two-dimensional (2D) and three-dimensional (3D) flutter modelling and analysis approaches and possible improvements in the structural system against flutter instability.

1.1. History and Development of Long-span Bridges

Cable-supported bridges such as suspension and cable-stayed bridges are common to connect great distances. The history of cable-supported bridges spans over two centuries. The first steel suspension bridge over the Jacob's Creek with 21m span was built in 1801 (Jurado and Albarracín, 2011) whereas the first cable-supported bridge composed of drawn iron wires was built in Geneva in 1823. Gimsing and Georgakis (2011); Myerscough (2013); Haifan (2011) describe the history and development of long-span cable-supported bridges around the world from the early 19th century.

The failure of the first Dryburgh Abbey Bridge in Scotland in 1818 due to wind is the first significant example when several of the stay connections failed during a storm. The Brighton Chain Pier, England (1836), the Tay Bridge, Scotland (1879) and the original Tacoma Narrows Bridge, U.S.A. (1940) are the most notable examples of long-span bridge failure where wind played a major role (Holmes, 2015). Gimsing and Georgakis (2011); Xu (2013) summarise several examples of wind-induced bridge failure in the past two centuries.

Although some bridges failed prior to the collapse of the original Tacoma Narrows Bridge due to wind actions, this was the failure that decreed bridge engineers to conduct scientific investigations of bridge aerodynamics. It was the third longest suspension bridge in the world with 853m main span. The deck started to oscillate with very large amplitudes in the vertical vibration modes at a wind speed of about 68km/h and around 45min later the motion changed to a torsional mode (Chen and Duan, 1999). This eventually led to failure of some overstressed members and the main span collapsed as shown in Figure 1. The mechanics behind dynamic wind-structure interaction was not fully understood at that time. Traditional practices of design were based on experience alone and the lack of understanding of the nature of wind mainly contributed to this (Gimsing and Georgakis, 2011). Billah and Scanlan (1991) explained the mechanism of the ultimate failure of the bridge and demonstrated physically, as well as mathematically, that the main cause was not the forced resonance but the self-excitation which is aerodynamically induced self-excitation. Scanlan (2002) and Miyata (2003) summarise a brief history of some developments in the field of bridge aerodynamics after the collapse of Tacoma Narrows Bridge and explain how the investigations have helped in the development of experimental and analytical methods for the prediction of long-span bridge response to the wind.

Today, the spans of the long-span suspension and cable-stayed bridges have been extended to new limits. The longest bridge in the world built at the end of the last century is the suspension bridge across the Akashi-Kaikyo Straits in Japan with a main span of 1991m and nearly 4km of overall length. Aerodynamic characteristics governed the design of this bridge (Holmes, 2015). The Russky Bridge in Russia (2012) is the world's longest cable-stayed bridge, with an 1104m long central span. Xu (2013) lists the top 10 longest suspension bridges and the top 10 longest cable-stayed bridges in the world. With the rapid growth of China's economy in the past couple of decades, more than 200 long-span bridges have been built. Ge and Xiang (2008a) provides history and new development of building bridges in China with the important structural properties and configurations of some bridges. Some examples of the major long-span bridges are listed in the Table 1 with their structural properties.

With the increase of bridge span length, the modern bridge structures are more flexible, lightweight and their structural characteristics require special treatment of aerodynamic analysis and design under wind action. Weight (2009) provides an informative and critical analysis of the Great Belt East Bridge in Denmark which is the third largest suspension bridge in the world. The proposed Messina bridge takes the Wind Engineering to its new limits with a main span of 3300m. There has been an extensive study made on the deck configuration

of the bridge. [Bartoli et al. \(2008\)](#) summarise wind tunnel tests, resulting in static wind coefficients, dynamic tests and numerical model time history analyses. An attempt has been made by [Ge and Xiang \(2007\)](#) to introduce a limit span length for suspension bridges and they later made an aerodynamic feasibility study of a 5000m span suspension bridge along with aerodynamic stabilisation.

1.2. Aeolastic Instability Phenomena

Dynamically flexible wind-sensitive structures, such as long-span bridges, oscillate under wind action. The motion of the structure generates aerodynamic forces which are known as motion-induced forces or self-excited forces. The overall system damping which is the combination of structural damping and the aerodynamic damping becomes negative at very high wind speeds. If the effective damping is not sufficient to dissipate the energy input by the wind, the amplitudes of the oscillating structure start to grow and diverge. The theoretical border between the decaying and diverging motions is known as the critical condition and the corresponding wind velocity is called the critical wind velocity. The aerodynamic damping remains positive below the instability limit which helps in reducing vibrations. The motion-induced part is insignificant for the short-span bridges and therefore there is no concern about aerodynamic instability ([Fujino and Siringoringo, 2013b](#)). The typical response amplitude of the bridge elements due to increasing wind speed is shown in Figure 2.

Static as well as the dynamic structural response will, in general, increase with increasing mean wind speed ([Strømmen, 2006](#)). However, in the case of aeroelastic instability, the response is rapidly increasing for even a small increase in the mean wind speed. Therefore, the aerodynamic phenomena for long-span bridges can be mainly classified into two types based on nature of amplitudes under wind action: limited-amplitude and divergent-amplitude wind-induced vibrations ([Simiu and Scanlan, 1996](#)). The former consists of Buffeting, Vortex-induced Vibrations (VIV), Rain Wind-induced Vibrations and Wake-induced Vibrations which are related to the serviceable discomfort, increased internal stresses and may cause fatigue in the bridge structure. The latter comprises of Galloping, Torsional Flutter and Coupled Flutter which can lead to structural instability. Instabilities resulting from the wind-structure interaction are known as aerodynamic or aeroelastic instabilities. The term aeroelastic emphasises the behaviour of deformed bodies, and aerodynamic emphasises the rigid bodies ([Chen and Duan, 1999](#)). Sometimes these two terms are used alternatively. The oscillation amplitudes of the bridge deck can build up until it results in the failure of the bridge. The collapse of the original Tacoma Narrows Bridge is an example which resulted in due to such an aeroelastic instability.

1.2.1. Classical Flutter

Flutter is a self-feeding aeroelastic phenomenon which is potentially destructive to long-span cable-supported bridges. The aerodynamic forces generated on the bridge deck couple with its motion and if the energy input by the aerodynamic forces at high wind speeds in an oscillation cycle becomes larger than that dissipated by the mechanical damping of the bridge structural system, the amplitude of vibration will grow. This increasing vibration will then amplify the aerodynamic forces, resulting in continuously growing self-excited forces and self-exciting oscillations. If this situation continues for some period of time, it can lead to the ultimate limit state of the bridge and ultimately to collapse.

The term classical flutter is originally applied to thin airfoils in the aircraft industry ([Xu, 2013](#)). Classical flutter is an aeroelastic phenomenon in which two-degrees-of-freedom (2DOF) of a structure, torsional motion and vertical bending, couple together in a flow-driven, unstable oscillation ([Simiu and Scanlan, 1996](#)). It is also known as 2DOF flutter, coupled flutter or simply flutter. The critical flutter condition occurs at the wind speed where energy input to the system is equal to the energy dissipated by the structural damping. The oscillatory motions of the contributing degree-of-freedom (DOF) in the structure couple to create a single frequency called the flutter frequency as shown in Figure 3. Flutter can happen in both laminar as well as in turbulent flows. When flutter occurs, the displacements become infinite by the linear theory. Vertical and torsional motion occur with a phase difference which is essential for the energy transfer from the wind to the structure. The mechanism of energy transfer in case of coupled flutter is described in Figure 4. The flutter phenomena will depend on phase lag or degree-of-coupling among the modes ([Fujino and Siringoringo, 2013b](#)). The smaller frequency separation

94 between the coupled modes increases the risk of flutter to occur (Dyrbye and Hansen, 1997). The self-excited motion can even cause flutter
95 on several DOF where higher modes also participate in the phenomenon.

96 The damping characteristics categorised flutter behaviour of the system into a soft or hard flutter. Chen and Kareem (2008) studied the rate
97 of change in modal damping with increasing wind speed, which is related to as soft or hard flutter. Hard-type flutter occurs by the rapid
98 growth of negative aerodynamic damping with increasing wind speed beyond the flutter limit whereas soft-type flutter happens where the
99 negative damping builds up slowly with increasing wind speed.

100 Flutter is destructive in nature with infinitely large amplitudes by the linear aerodynamic theory. In reality, the aerodynamic nonlinear
101 effects take place which lead to Limit Cycle Oscillation (LCO) where the amplitudes are limited. This difference can be visualised in
102 Figure 5.

103 1.2.2. Galloping

104 Galloping is a single-degree-of-freedom (SDOF) large-amplitude aeroelastic oscillation in the transverse direction to wind. It occurs as a
105 result of negative aerodynamic damping in the cross wind direction. The amplitudes of oscillation can reach up to many times the section
106 depth. It is sometimes also called as translational galloping, cross-wind galloping or bending flutter. Galloping is usually experienced by
107 D-shaped cross-section, square section (at zero angle of attack) and circular section with some accretion such as ice-coated transmission
108 line or guyed cables. It can also happen for a bridge with bluff cross-section and a lightweight superstructure even for a shorter span.
109 The cross-section with larger aspect ratio (depth/width) has a higher tendency to gallop which is known as soft galloping, whereas deeper
110 sections require initial perturbation that leads to hard galloping.

111 Galloping occurs due to change in the effective angle of attack of the vertical or the torsional motion of the structure. It can be evaluated
112 by checking signs of the time-averaged static wind coefficients for lift and moment at zero angle of attack. A negative slope of static lift
113 (or moment) coefficient usually indicates a tendency for galloping. The negative lift coefficient implies that the section will be pushed
114 upward resulting in a divergent response or galloping (Chen and Duan, 1999). Galloping depends mainly on the quasi-steady behaviour of
115 the structure and therefore the mechanism of galloping can be explained by the quasi-steady aerodynamic theory. The analytical solution
116 for Galloping can be obtained from Dyrbye and Hansen (1997). The difference between the critical velocity predicted for galloping by the
117 linear and non-linear aerodynamic theory is insignificant (Fujino and Siringoringo, 2013b); however, the nonlinear aerodynamic theory
118 provides also amplitude and frequency. Some studies utilised wind tunnel experiments to explain different aspects related to understanding
119 the mechanism of galloping of rectangular cylinders with different aspect ratios (Washizu *et al.*, 1978) and galloping behaviour of Yadagawa
120 Bridge in Japan (Ge *et al.*, 2002).

121 1.2.3. Torsional Flutter

122 Torsional flutter is a rotational SDOF aeroelastic instability. It is sometimes also referred to as rotational galloping, torsional galloping
123 or stall flutter in aeronautical terminology. The mechanism of torsional flutter is related to aerodynamic damping of rotational motion.
124 Torsional flutter occurs when the total damping (mechanical plus aerodynamic) of the system in torsional motion becomes zero. The vertical
125 mode of vibration of a structure has some contribution; however, the torsional action is more critical by far (Fujino and Siringoringo, 2013b).
126 It occurs in long-span bridges with bluff deck sections such as H-shape, a rectangular section with larger aspect ratio, or truss sections,
127 whereas streamlined sections are more favourable for the classical flutter.

128 Blevins and Iwan (1974) studied the mechanism of torsional flutter by the linear and nonlinear forms of quasi-steady theory. Nakamura
129 and Mizota (1975); Nakamura (1979) explained the torsional flutter by the theoretical and experimental study as an unsteady phenomenon.
130 Nakamura (1979) also described torsional flutter by performing experimental forced vibration tests and gave the unsteady aerodynamic
131 lifts and moments. Moreover, Washizu *et al.* (1980) studied rectangular cylinders with wind tunnel experiments and highlighted the effect

of aspect ratio on the aeroelastic instability phenomena in the vicinity of the critical speed. The aerodynamic force with the same frequency as that of Strouhal number St has a negative effect and can excite torsional motion in the deck (Zhou *et al.*, 2006). Matsumoto *et al.* (1997, 2008d); Matsumoto (2009) described the torsional flutter mechanism of rectangular and H-shaped cylinders (Tacoma Narrows Bridge deck) and investigated the role of vortex excitation and Karman vortex excitation on the torsional flutter behaviour. The equations to solve torsional flutter problem can be found in Dyrbye and Hansen (1997). Apart from that, the phenomenon of torsional flutter has been studied by several experimental (Daito *et al.*, 2002) as well as numerical (Morgenthal *et al.*, 2005) approaches.

1.2.4. Aerostatic Divergence

Static divergence, sometimes also referred to as torsional divergence, is a phenomenon of losing torsion stiffness at a relatively high wind speed. It presents a static type of instability phenomenon where the structure does not oscillate. Divergence problem involves a combination of torsional divergence and lateral buckling. At a critical wind speed for torsional divergence, the bridge deck may experience out-of-plane buckling under the action of a drag force or torsionally diverge under a wind-induced moment that increases with the increase in geometric twist angle (Chen and Duan, 1999). Matsumoto *et al.* (2010), additionally, classified torsional divergence into static SDOF torsional divergence and dynamic 2DOF torsional divergence.

2. Motion-induced Aerodynamic Forces

2.1. Analytical Models for Flat Plate

Studies of the flutter phenomenon originated from the classical theory of aeroelasticity within Aeronautical Engineering. This discipline combines the theories of Fluid Dynamics and Deformable Solid Mechanics (DSM) by imposing certain boundary condition for the Fluid-structure Interaction (FSI). Historically, the earliest studies about this phenomenon are for aircraft wings in the beginning of the 20th century. The description of the self-excited forces is based on a thin airfoil which can be described as a flat plate with half-chord b with infinitesimal thickness, as depicted in Figure 6, are the basis of their formulation for bluff bodies. All the analytical flat plate models have a set of common assumptions: the fluid is inviscid and incompressible; the velocity field is irrotational i.e. they are based on the potential flow and small angles of attack; the strip assumption applies regarding the finite-span effect. This means that the aerodynamic force at any chord-wise section is the same as if that section was situated in a 2D flow. The study does not aim to cover all aspects of the development on flat plate aerodynamics, rather giving a physical insight of the problem in order to be able to introduce the semi-empirical models for bluff bodies. Dowell (2004); Fung (2002); Bisplinghoff *et al.* (1996); Jurado and Hernández (2000) gave an extended attention on the flat-plate problems.

Subjecting the flat plate to a steady laminar flow with a direction rotated by angle α with respect to the chord of the flat-plate, a lift force F_L and a moment F_M are acting on the flat plate. These steady forces are a consequence of the distribution of the strength of vorticity around the body, the fluid density and the wind velocity. Considering a step rotation of the flat plate, a vortex is shed smoothly downstream from a single point of the trailing edge (also known as the Kutta-Joukowski condition). The circulation due to this free vortex has its counterpart on the flat plate of the same magnitude in order to satisfy zero circulation condition as assumed in the potential flow theory, which results in a force. The strength of the vorticity is time-dependent which results in an asymptomatic rise in the lift force towards the steady state forces. This was initially studied by Wagner (1925) who described the non-stationary lift force as a function of the reduced time by the so-called Wagner indicial function $\phi(\tau)$ in order to describe the circulatory part of the unsteady lift and moment (Fung, 2002):

$$(1a) \quad F_L = \rho \pi U_\infty^2 (b\alpha' + h'') + 2\pi \rho b U_\infty^2 \left[\int_{-\infty}^{\tau} \phi(\tau - \tau_1) \frac{d}{d\tau_1} \left(\alpha(\tau_1) + \frac{h'(\tau_1) + 0.5b\alpha'(\tau_1)}{U_\infty} \frac{U_\infty}{b} \right) d\tau_1 \right],$$

$$(1b) \quad F_M = -\rho b^2 \pi U_\infty^2 \left(\frac{1}{2} \alpha' + \frac{1}{4} \alpha'' \right) + \pi \rho b^2 U_\infty^2 \left[\int_{-\infty}^{\tau} \phi(\tau - \tau_1) \frac{d}{d\tau_1} \left(\alpha(\tau_1) + \frac{h'(\tau_1) + 0.5b\alpha'(\tau_1)}{U_\infty} \frac{U_\infty}{b} \right) d\tau_1 \right].$$

where h is the vertical displacement, ρ is the mass density of the air, b is the half chord ($b = B/2$), U_∞ is the oncoming wind velocity, respectively. The reduced non-dimensional time is defined as $\tau = U_\infty t/b$ and the prime on h and α is the differentiation with respect to

the non-dimensional time. In addition to the circulatory forces, there are apparent mass forces due to the inertial effect related to the portion of fluid surrounding the body that moves with it which are not associated with the creation of vorticity and are of minor importance in the bending-torsion flutter. In Figure 7a, the approximation of the Wagner function by Jones (1940) is shown, from which it can be seen that as $\tau \rightarrow \infty$, the indicial function $\phi \rightarrow 1$ which is the quasi-steady state. If the indicial function is considered as a step function, the quasi-steady representation can be obtained (c.f. Section 2.2.3). The quasi-steady model neglects the rise time of the indicial function or sometimes referred to as the fluid memory, by mapping the instantaneous state of the system at the current time into the corresponding state at an infinite time, such that the vortex has been convected far from the body for evaluating the flow-induced forces. The apparent mass forces are usually neglected when the quasi-steady assumption applies (Fung, 2002).

For a flat plate oscillating harmonically with a certain frequency, each of the vortices in the wake will influence the forcing on the flat plate i.e. the forcing will be dependent on the history of the motion or the so-called fluid memory effect. Theodorsen (1935) gave an exact solution of a harmonically oscillating flat plate for all reduced frequencies:

$$(2a) \quad F_L = \rho b^2 \pi (U_\infty \dot{\alpha} + \ddot{h}) + 2\pi \rho b U_\infty^2 C(k) \left(\alpha + \frac{\dot{h} + 0.5b\dot{\alpha}}{U_\infty} \right),$$

$$(2b) \quad F_M = -\rho b^2 \pi \left(U_\infty \frac{b}{2} \dot{\alpha} + \frac{b^2}{8} \ddot{\alpha} \right) + \pi \rho b^2 U_\infty^2 C(k) \left(\alpha + \frac{\dot{h} + 0.5b\dot{\alpha}}{U_\infty} \right),$$

where $C(k) = F(k) + iG(k)$ is the Theodorsen's circulation function which depends on the reduced frequency $k = b\omega/U_\infty$ depicted in Figure 7b. It describes the non-stationary contribution due to FSI. It can be seen for the low reduced frequencies $k \rightarrow 0$ the real part $F \rightarrow 1$ and the imaginary part $G \rightarrow 0$, which corresponds to the quasi-steady state. Garrick (1938) pointed out that Theodorsen's and Wagner's functions constitute a Fourier transform pair. In Peters (2008) an overview is given of the recent developments in the unsteady airfoil theory including closed-form solution and the solution using finite state models (c.f. Section 2.2.2).

2.2. Semi-empirical Models for Bluff Bodies

There are four important actions to induce aerodynamic forces on bluff bodies namely: static mean wind, vortex-induced excitation, the interaction between the wind and structural motion and in the case of turbulent inflow conditions additional aerodynamic force. This will correspondingly result in static mean wind force, vortex-excited force, motion-induced force and buffeting force, respectively (Ge and Xiang, 2008c), where the self-excited forces play the most important role in the bridge flutter phenomenon. The FSI in cases of general non-streamlined bluff bodies is a complex phenomenon and to date, to the authors' knowledge, there is no general circulatory theory which can describe the motion-induced forces analytically, analogous to those for a flat plate based on the potential flow theory. This is a consequence of the flow separation and reattachment which could occur at multiple points on the solid boundary and it contradicts the assumption for the shedding of vortices from a single point. Therefore a significant amount of research has been associated with developing a semi-empirical model dependent on the motion and its time derivatives and additional empirically based aerodynamic derivatives. For a 2DOF section model, the system could be described (Tamura and Kareem, 2013) generally by:

$$(3a) \quad m\ddot{h} + 2m\xi_h\omega_h\dot{h} + m\omega_h^2 h = F_L(h, \dot{h}, \ddot{h}, \alpha, \dot{\alpha}, \ddot{\alpha}),$$

$$(3b) \quad I\ddot{\alpha} + 2I\xi_\alpha\omega_\alpha\dot{\alpha} + I\omega_\alpha^2 \alpha = F_M(h, \dot{h}, \ddot{h}, \alpha, \dot{\alpha}, \ddot{\alpha}),$$

where m and I are the mass and mass moment of inertia, ξ_h and ξ_α are the damping ratios and ω_h and ω_α are the natural circular frequencies for the heave and pitch direction, h and α are the vertical displacement and rotation, F_L and F_M are the lift force and moment, respectively. In the case of bridge decks, the apparent mass terms dependent on the acceleration of the structures are usually neglected (Wu and Kareem, 2013). Considering a whole structure, the modal equation of motions are:

$$(4) \quad \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{C}_{ae}\dot{\mathbf{x}}(t) + \mathbf{K}_{ae}\mathbf{x}(t).$$

where $\ddot{x}$ is the acceleration, $\dot{x}$ is the velocity and x is the displacement, respectively. The left-hand side of Eq. (4) represents structural properties of the bridge deck including modal mass $\mathbf{M}$, modal damping $\mathbf{C}$ and modal stiffness $\mathbf{K}$, whereas the right-hand-side of equations describe the unsteady motion-induced aerodynamic force which can be divided into damping related and stiffness related terms. The term motion-induced indicates that the force acting on the deck not only depends on the instantaneous value but also on the history of motion. Even if there is no coupling on the structural part, a coupling on the aerodynamic side could occur in the aerodynamic damping $\mathbf{C}_{ae}$ and aerodynamic stiffness $\mathbf{K}_{ae}$ matrices (Chen and Duan, 1999).

Historically, Duncan and Frazer (1928) first offered 2D form for the flutter equations, followed by Bleich (1948) with a series of publications for truss bridges; however, the major milestone in bridge flutter was set by Scanlan and Tomko (1971) who formulated the problem by expressing the motion-induced forces based on Theodorsen's theory and experimentally obtained aerodynamic derivatives, the so-called Scanlan derivatives, linearly dependent on the motion of the structure.

2.2.1. Scanlan and Equivalent Representations

The full expression of Scanlan's model is expressed as a linear function of the motion considered in Theodorsen theory multiplied by aerodynamic derivatives. Scanlan and Tomko (1971) introduced the self-excited forces as in Figure 8 by using aerodynamic derivatives obtained from an experimental approach:

$$(5a) \quad F_L = \frac{1}{2} \rho U_\infty^2 B \left[K H_1^* \frac{\dot{h}}{U_\infty} + K H_2^* \frac{B \dot{\alpha}}{U_\infty} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} + K H_5^* \frac{\dot{p}}{U_\infty} + K^2 H_6^* \frac{p}{B} \right],$$

$$(5b) \quad F_M = \frac{1}{2} \rho U_\infty^2 B^2 \left[K A_1 \frac{\dot{h}}{U_\infty} + K A_2 \frac{B \dot{\alpha}}{U_\infty} + K^2 A_3 \alpha + K^2 A_4 \frac{h}{B} + K A_5 \frac{\dot{p}}{U_\infty} + K A_6 \frac{p}{B} \right],$$

$$(5c) \quad F_D = \frac{1}{2} \rho U_\infty^2 B \left[K P_1^* \frac{\dot{p}}{U_\infty} + K P_2^* \frac{B \dot{\alpha}}{U_\infty} + K^2 P_3^* \alpha + K^2 P_4^* \frac{p}{B} + K P_5^* \frac{\dot{h}}{U_\infty} + K^2 P_6^* \frac{h}{B} \right],$$

where F_D is the drag force, p is the lateral displacement, $K = B\omega/U_\infty$ is the reduced frequency, ω is the frequency of oscillation and H_i^* , A_i^* and P_i^* (with $i = 1..6$) are Scanlan's derivatives, respectively. These aerodynamic derivatives for bluff sections cannot be computed analytically; however, it is possible to obtain them by experimental wind tunnel tests or through CFD simulations which also discussed in Section 2.5.1.

The motion-induced aerodynamic forces are based on two assumptions (Ge and Xiang, 2009b). Firstly, the displacement of the structure follows harmonic vibration and can be represented with 18 aerodynamic derivatives neglecting the apparent mass for the acceleration term. Secondly, if the motion-induced forces are considered to have a non-linear relationship with structural motion, then these expressions would include the second-order or even high-order terms of the structural motion. Scanlan's model has been widely used for the solution of 2D and 3D aeroelastic instability problems in the field of bridge aerodynamics due to its applications to different types of bridge cross sections. Scanlan and Tomko (1971) initially studied free vibration methods to identify aerodynamic derivatives (H_i^* , A_i^* with $i = 1..3$) by using experiments for 5 different deck shapes and studied the effect of the change in depth of H-shape section. The reason for using 3 aerodynamic derivatives was the direct relationship with the flat-plate theory which is based only on vertical motion, angular rotation and angular velocity in the circulatory terms. Scanlan (1978a,b) utilised (H_i^* , A_i^* , P_i^* with $i = 1..3$) aerodynamic derivatives for the linear dynamic analysis of bridge flutter for bridges having lateral vibration modes along with the energy considerations involved in the assessment of aerodynamic stability. Jain et al. (1996b,a) made use of all 6 P_i^* aerodynamic derivatives in the aerodynamic analysis.

In the field of bridge aerodynamics, comparing aerodynamic characteristics utilising the aerodynamic derivatives is a common and widely accepted approach. Various conventions have been developed and are used for this propose based on the same assumptions as Scanlan's. The earlier ones are developed by Miyata et al. (1975). Zasso (1996) gave a representation of the motion-induced forces with the advantage of a better description in the low-frequency range where Scanlan's tend to become zero. The coefficients (H_i^* , A_i^*) are the real and the

imaginary parts of the transfer functions between the non-dimensional forces and the non-dimensional displacements. Motion-induced aerodynamic forces are mathematically described in real number or complex number expressions and the flutter analysis performed using either of these lead to the same numerical results. Starossek (1992) represented the motion-induced forces in the complex form with the advantage in the solution method by solving a linear eigenvalue problem with which the complex flutter mode shape is obtained without additional effort.

In the aerodynamic study for the Normandy Bridge in France, a so-called Küssner notation was used, with the aerodynamic derivatives are known as Küssner coefficients (k_i^a , k_i^b , m_i^a , m_i^b with $i = 1, 2$) (Szechenyi, 1973). Zasso (1996) and Borri and Höffer (2000) summarised some of these conventions and highlighted their merits and disadvantages. Jensen (1997) compared the Scanlan's derivatives with the ones developed for Marine Engineering usage based on the relative position of the bridge deck utilising the velocity and acceleration terms. Some of these conventions are summarised in Table 2.

2.2.2. Impulse and Indicial Function Representations

The Scanlan's representation of the motion-induced forces includes time and frequency dependent terms. In order to conduct a pure time-domain analysis, the aeroelastic forces should be formulated without the frequency dependent terms. This is realised by indicial and impulse functions or sometimes referred to as approximation kernels. In fact, they are closely related to each other and as pointed out in Wu and Kareem (2014b), the impulse function is the derivative of the indicial function w.r.t. time, with an exception at $t = 0$. The formulation w.r.t. indicial function is by expanding the Wagner formulation of non-stationary forces for a bluff body and neglecting the non-circularity terms related to apparent mass (Caracoglia and Jones, 2003b):

$$(6a) \quad F_L = -\frac{1}{2}\rho U_\infty^2 BC'_L \int_{-\infty}^{\tau} \left(\Phi_{L\alpha}(\tau - \tau_1) \alpha'(\tau_1) + \Phi_{Lh}(\tau - \tau_1) \frac{2h''(\tau_1)}{B} + \Phi_{Lp}(\tau - \tau_1) \frac{2p''(\tau_1)}{B} \right) d\tau_1,$$

$$(6b) \quad F_M = \frac{1}{2}\rho U_\infty^2 B^2 C'_M \int_{-\infty}^{\tau} \left(\Phi_{M\alpha}(\tau - \tau_1) \alpha'(\tau_1) + \Phi_{Mh}(\tau - \tau_1) \frac{2h''(\tau_1)}{B} + \Phi_{Mp}(\tau - \tau_1) \frac{2p''(\tau_1)}{B} \right) d\tau_1,$$

$$(6c) \quad F_D = \frac{1}{2}\rho U_\infty^2 BC'_D \int_{-\infty}^{\tau} \left(\Phi_{D\alpha}(\tau - \tau_1) \alpha'(\tau_1) + \Phi_{Dh}(\tau - \tau_1) \frac{2h''(\tau_1)}{B} + \Phi_{Dp}(\tau - \tau_1) \frac{2p''(\tau_1)}{B} \right) d\tau_1,$$

where, $\Phi_{Q,\alpha}$, $\Phi_{Q,h}$, $\Phi_{Q,p}$ (with $Q = L, M, D$) are indicial functions describing the dimensionless lift, moment and drag force due to step change of α , h , p , respectively. The motion-induced forces were initially introduced only for lift and moment by Scanlan *et al.* (1974) for bridge decks. Lin and Ariaratnam (1980); Lin and Li (1993) simplified this formulation considering only rotational motion in order to study the torsional stability in turbulent wind. These were then extended by Scanlan (1993) for the drag force based on torsional and lateral motion and the final form was used after Jain *et al.* (1996b) studied the effect of the horizontal motion on the self-excited forces.

The phenomenon of “overshooting” occurs for bluff bodies in which the indicial function has a higher value than unity in the rise time and then drops. This is a consequence of the flow separation and it is addressed by Scanlan (2000); Wu and Kareem (2014b). Comparing to the original flat plate theory, this form does not incorporate the indicial function dependent on the angular acceleration term α'' . These terms were originally included in the notations by Scanlan *et al.* (1974); however, Lin and Yang (1983) Bucher and Lin (1988, 1989) described this as redundant relation, given that the angular velocity α' incorporates information about the angular acceleration α'' in the interval $[-\infty; \tau]$. Changing the order of integration by partial integration, the limits of the integral change from ∞ to 0; thus, numerical integration

is in order. The impulse function formulation was initially given by [Lin and Yang \(1983\)](#) and then by [Bucher and Lin \(1988, 1989\)](#):

$$(7a) \quad F_L = -\frac{1}{2}\rho U_\infty^2 \int_{-\infty}^t \left(I_{L\alpha}(t-\tau_1)\alpha(\tau_1) + I_{Lh}(\tau-\tau_1)h(\tau_1) + I_{Lp}(\tau-\tau_1)p(\tau_1) \right) d\tau_1,$$

$$(7b) \quad F_M = \frac{1}{2}\rho U_\infty^2 \int_{-\infty}^t \left(I_{M\alpha}(t-\tau_1)\alpha(\tau_1) + I_{Mh}(\tau-\tau_1)h(\tau_1) + I_{Mp}(\tau-\tau_1)p(\tau_1) \right) d\tau_1,$$

$$(7c) \quad F_D = \frac{1}{2}\rho U_\infty^2 \int_{-\infty}^t \left(I_{D\alpha}(t-\tau_1)\alpha(\tau_1) + I_{Dh}(\tau-\tau_1)h(\tau_1) + I_{Dp}(\tau-\tau_1)p(\tau_1) \right) d\tau_1,$$

where, $I_{Q,\alpha}, I_{Q,h}, I_{Q,p}$ (with $Q = L, M, D$) are aerodynamic impulse response functions describing the lift, moment and drag force evolution for a certain time period due to change of the motion at a certain time instant α, h, p , respectively. The determination of the impulse and indicial function is discussed in the Section 2.5.2. [Wu and Kareem \(2014b\)](#) compared both the indicial and impulse function approaches for bluff body aerodynamics on the theoretical, experimental and numerical basis, concluding that they are essentially very similar if not the same.

2.2.3. Quasi-steady Representations

The quasi-steady theory for bluff bodies essentially does not differ from the one for flat plates. This model does not consider the fluid memory effect; however, it offers a static nonlinear relationship between the instantaneous angle of attack and the flow-induced forces. Ignoring the apparent mass for laminar flow, the self-excited forces can be represented as ([Kovacs et al., 1992](#); [Borri and Costa, 2004](#)):

$$(8a) \quad F_L = -\frac{1}{2}\rho U_r^2 B C_L(\alpha_e),$$

$$(8b) \quad F_M = \frac{1}{2}\rho U_r^2 B^2 C_M(\alpha_e),$$

$$(8c) \quad F_D = \frac{1}{2}\rho U_r^2 B C_D(\alpha_e),$$

where,

$$(9a) \quad U_r = \sqrt{(U_\infty - \dot{p})^2 + (\dot{h} + m_i B \dot{\alpha})^2},$$

$$(9b) \quad \alpha_e = \alpha + \left(\frac{\dot{h} + m_i B \dot{\alpha}}{U_\infty - \dot{p}} \right).$$

With the effective angle of attack α_e and relative velocity U_r , the fluid-structure interaction is described. The major challenge in this model is to determine the parameter m_i for $i = L, M, D$, which describes the distance between the aerodynamic and stiffness centre. This coefficient depends on the pressure distribution, as well as the separation of the fluid from the body and in fact, is nonlinear and frequency dependent; therefore, there are challenges in its determination. [Diana et al. \(1993, 2013\)](#) determined this coefficient based on the flutter derivatives at a certain reduced velocity, for which the quasi-steady assumption holds as depicted in Table 3. [Wu and Kareem \(2013\)](#) made extended discussion on the importance of this parameter in determining equivalent steady-state for applying the quasi-steady model. Linearising, taking quarter chord distance for m ($m = 0.25$), the quasi-steady model for a flat plate is obtained. In order to take into account the fluid memory term, [Diana et al. \(1992, 1993\)](#) introduced additional factors K_D, K_L, K_M for the static wind coefficients to develop the corrected quasi-steady theory:

$$(10a) \quad C_D(\alpha_e) = C_D(\alpha_0) + \int_{\alpha_0}^{\alpha_e} K_D d\alpha,$$

$$(10b) \quad C_L(\alpha_e) = C_L(\alpha_0) + \int_{\alpha_0}^{\alpha_e} K_L d\alpha,$$

$$(10c) \quad C_M(\alpha_e) = C_M(\alpha_0) + \int_{\alpha_0}^{\alpha_e} K_M d\alpha.$$

According to Diana *et al.* (1993, 2013) these three parameters can be obtained for high reduced velocities from the flutter derivatives or through wind-tunnel testing (Falco *et al.*, 1992; Diana *et al.*, 1992, 1995). Table 3 shows the relation between Scanlan's representation and quasi-steady theory for a selected reduced velocity. Another modified quasi-steady model was developed by Øiseth *et al.* (2010) for a case study in Norway's Hardanger Bridge. This model was based on the frequency-independent interpolation of the flutter derivatives at the structural frequency and it seems to be corresponding well with the other models for the specific case study.

2.2.4. Recent Developments

Recently, Diana *et al.* (2008) developed a new concept of modelling the aerodynamic forces based on the hysteresis behaviour of the force and moment coefficient with respect to the dynamic angle of attack for different reduced velocities. The challenge, in this case, is to formulate a rheological element which could encapsulate the unsymmetrical nonlinear hysteresis. The model is based on several independent mechanical oscillators in a parallel configuration, whose parameters are obtained through the concept of an aerodynamic force transfer function. In Wu and Kareem (2011), a nonparametric model was developed based on an input-output black box, Artificial Neural Network (ANN) framework was presented in order to describe the hysteretic behaviour. Although promising results are shown for the coupled gust-induced and motion-induced forces, the model is still not applied for the computation of the flutter velocity. However, the same concept of ANN was used by Wu and Kareem (2013) for approximating directly the aerodynamic hysteresis w.r.t. the angle of incidence in the modified hybrid model, where the flutter velocity was obtained. To further investigate the aerodynamic nonlinearities, Wu and Kareem (2014a) used the Volterra series as approximation kernels to simulate the nonlinear system with fading memory term due to the viscosity. With this formulation, the nonlinear system is modelled as an infinite sum of multidimensional convolution integrals of increasing order, which directly correlates to the unsteady aerodynamic forces. Comparison of the modelling schemes and discussions on the topic can be found in Scanlan (1993, 2000, 2001); Chen and Kareem (2002); Wu and Kareem (2013).

2.3. Numerical Methods

The aerodynamic behaviour for the design of long-span bridges is traditionally studied through wind tunnel tests; however, these tests are cost-expensive and require time for planning, actual testing and may be prohibitive at the design stage. Whilst detailing can be easy, a small modification of the global shape of the section requires a new model. Computational Fluid Dynamics (CFD) techniques have gained much attention and development in the last two decades and are used as alternative beside the analytical and experimental methods. The techniques are based on the numerical discretization of the Navier-Stokes (N-S) equations and by introducing a shape similar to a bridge deck immersed in the flow allows to determine the force coefficients, pressure distribution, aerodynamic derivatives and flow visualisation, which leads to the insightful understanding of the aerodynamic phenomena. However, still, it has limitations with the complexity and nature of 3D bridge structures (Larsen, 1997). CFD is more accessible than wind tunnel testing for the aerodynamic analysis of bridge deck geometry and checking the performance of structure as pointed out by Ge and Xiang (2008a). Blocken (2014) provides a perspective on the past, present and future of CFD application in Wind Engineering or the so-called field Computational Wind Engineering (CWE).

There are various methods for the discretization of the flow field; however, the most commonly used in the field of bridge aerodynamics are the Finite Element Method (FEM), Finite Volume Method (FVM) and Discrete Vortex Method (DVM). The FVM and FEM methods are Eulerian mesh-based methods with the main difference that the FEM also models the flow between nodal points utilising the shape functions, whilst the DVM is Lagrangian mesh-free. Earlier studies based on the FEM (Frandsen, 2004) did not include turbulence models for the closure of the N-S equations and although the computations yielded comparable results, a remark for further elaborate studies was made. Most recent results based on the FEM and FVM include turbulence models such as the Reynolds-Average Navier-Stokes (RANS) models based on time averaging of the flow, Large Eddy Simulation (LES) based on spatial averaging of the flow and hybrid models such as Detached Eddy Simulation (DES) and Hybrid RANS-LES models. An early comparison was made by Bruno and Khris (2003) for 2D numerical models using the FVM with RANS and 2D LES as turbulence models including parametric studies for the computational parameters, where emphasis was given on the importance of modelling of the deck equipment. Recently, Brusiani *et al.* (2013) made a

comparison of the 2D two-equations RANS based models ($\kappa - \epsilon$, $\kappa - \omega$ and $\kappa - \omega$ SST) for the determination of the flutter derivatives, concluding that all of them yielded comparable results. The difference between 2D and 3D simulations w.r.t. flutter derivatives were studied in [Bai *et al.* \(2010\)](#) and [Ketong and Aiping \(2014\)](#) using the DES turbulence model with FVM. The first reference made a remark of the importance of the 3D CFD computations for complex geometries; however, comparable results were obtained for a generic streamlined deck between the 2D and 3D simulations. The DVM has gained much attention in the 1990s to be used in the field of bridge aerodynamics. The success of the simulations is attributed to the bluff nature of the cross-sections and to the 2D nature of flow around bridge girders ([Walther, 1994](#)). It offers significantly lower numerical diffusion compared to mesh methods and high computational efficiency; however, various a-priori numerical parameters and insufficient number of discrete vortices might be seen as a disadvantage which results in a model, rather than a direct numerical simulation. However, the latter could be reduced by introducing the Vortex-In-Cell technique, formulating a background grid for keeping evenly spaced particles. It should be noted that this grid is for practically averaging the vorticity and it does not introduce any numerical diffusion ([Morgenthal, 2002](#)). It has been used to study flow around a flat plate as well as other bluff bridge deck cross sections to show the applicability and efficiency of the method and to help understand mechanisms related to the phenomena of aeroelastic instability ([Taylor and Vezza, 1999](#); [Larsen and Walther, 1997a](#); [Morgenthal, 2005, 2006](#); [Morgenthal and Walther, 2007](#); [Morgenthal *et al.*, 2014](#); [Abbas and Morgenthal, 2014a, 2013c,a, 2014b](#); [Abbas, 2016](#)).

The aim of the study is not to offer a critical in-depth comparison of the various CFD techniques, rather present them as an alternative method in the procedure for determining the critical flutter limit. Furthermore, it offers a list of studies presented in [Tables 4 and 5](#) for further reference. An insightful comparison of the numerical methods is done by [Ge and Xiang \(2009c\)](#) where the three methods were compared for a flat plate and bluff bodies, all resulting in comparable results.

2.4. Experimental Methods

2.4.1. Full-scale Monitoring

Wind tunnel tests are the basis for analysis and design of long-span bridges. However, small-scale experiments have limitations in predicting the full-scale behaviour due to scale effects such as Reynolds number and the structural damping. The modelling parameters can only be validated through full-scale measurements which are crucial for bridge aerodynamics. Full-scale measurements are seldom performed for long-span bridges. [Frandsen \(2001\)](#) utilised full-scale measurements on the Great Belt Suspension Bridge and provided several other references of full-scale measurements on different bridges around the world. Moreover, full-scale measurements have been used to validate the analytical response prediction ([Jones and Ozkan, 2002](#)) and to extract aerodynamic derivatives ([Nikitas *et al.*, 2008](#)). The full-scale aeroelastic behaviour of bridges is still not completely understood and is an active research topic.

2.4.2. Wind Tunnel Testing

The tests in the wind tunnel are commonly carried out either on section models, taut-strip models or full aeroelastic models. These tests require scaled models representing the structure by ensuring certain similarity laws. Wind tunnel tests for the Akashi Kaikyo Bridge by [Miyata and Yamaguchi \(1993\)](#) and Great Belt East Bridge by [Larsen \(1993\)](#) are notable examples of 3D aeroelastic bridge models. Moreover, [Sato *et al.* \(2002\)](#) and [Yoshizumi and Inoue \(2002\)](#) describe 2D and 3D flutter analyses of full aeroelastic model tests for long-span bridges.

Section models are more commonly used for defining static wind loads ([Davenport and King, 1984](#)) and measuring aerodynamic derivatives of the section. These tests are also useful to study the aeroelastic instability characteristics of the deck by making use of the wind tunnel experiments on dynamically mounted section models. [Matsumoto \(1996\)](#) studied aerodynamic derivatives for prisms and their dependence on each other and categorised low-speed torsional flutter, high-speed torsional flutter and coupled flutter from the unsteady pressure measurement under the SDOF heaving/torsional forced vibration. [Table 6](#) provides some examples of wind tunnel tests performed on different structures.

2.5. Extraction of Aerodynamic Derivatives and Rational Function Approximation

2.5.1. Free and Forced Vibration Techniques

The aerodynamic unsteady forces or motion-induced forces are assumed to be related to the aerodynamic derivatives and their linear combination of displacements and their first time derivatives. There are two main approaches (Tamura and Kareem, 2013; Xu, 2013) to determine the aerodynamic derivatives for a section: first with direct measurements of aerodynamic force components by strain measurements on sectional model when the section is given some forcing (Matsumoto *et al.*, 1997) and second to calculate the force indirectly by measuring unsteady surface pressure and phase difference from the flow-induced motion of the model (Scanlan and Tomko, 1971). The direct measurement requires usually more complex experimental set-ups.

The motion of the structure in the flow induces aerodynamic forces which are characterised by aerodynamic derivatives. These aerodynamic derivatives are commonly measured from wind tunnel tests by using different methods such as free vibration, forced vibration and buffeting tests. In forced vibration tests the scaled deck section is forced to move with specific displacement and velocities (or angular rotations and angular velocities) and the aerodynamic forces are measured from which aerodynamic derivatives are extracted. Free vibration tests are more common where the decay in system damping and the change in system stiffness is measured when a deck section model is given some initial displacement. The buffeting tests are simple but the measurements need to be treated to remove buffeting forces considered as noise which require more advanced system identification techniques.

Some advancement has been made in the efficacy of a customarily used identification procedure (Chen and Kareem, 2004), as well as using ANN algorithms, to predict flutter derivatives (Chen *et al.*, 2008) and equivalent Theodorsen functions (Matsumoto *et al.*, 1993). The free vibration tests are usually performed with coupled 2DOF motion in heave and pitch (Gu *et al.*, 2000; Brownjohn and Jakobsen, 2001; Zhu *et al.*, 2007; Xu *et al.*, 2014); however, it becomes more challenging for 3DOF systems (Chen *et al.*, 2002; Chowdhury and Sarkar, 2003; Xu *et al.*, 2012). The response is treated with the general least-squares theory (Chen *et al.*, 2002), the iterative least squares method (Chowdhury and Sarkar, 2003), modified unified-least-squares method (Bartoli *et al.*, 2009; Xu *et al.*, 2012) or weighting ensemble least-square method (Li *et al.*, 2003) to compute the aerodynamic derivatives. Figure 9 shows a least-squares fit on a force time history in a forced vibration simulation. The classical forced vibration tests are conducted on decoupled motion to compute aerodynamic derivatives. However, this decoupled motion provides the aerodynamic derivatives where the direct components of aeroelastic motion are dominant than the interactive components (Xu *et al.*, 2014). In addition to the uncoupled ones, coupled forced vibration techniques are also used to determine aerodynamic derivatives (Wu *et al.*, 2006; Haung and Liao, 2011; Neuhaus *et al.*, 2009; Larsen *et al.*, 2011). Qin and Gu (2004); Gu and Qin (2004); Boonyapinyo and Janesupasaeree (2009); Boonyapinyo *et al.* (2009) used the covariance-driven Stochastic Subspace Identification (SSI-COV) technique which treats the buffeting forces and responses as input in the analysis rather than noise. Moreover, Boonyapinyo and Janesupasaeree (2010) also introduced the data-driven Stochastic Subspace Identification (SSI-DATA) technique. Some efforts have been made to use very small-scale wind tunnel testing to obtain aerodynamic derivatives (Piña and Caracoglia, 2009; Hunyadi, 2009). Some wind tunnel tests performed on different structures are summarised in Table 6 using free, forced and buffeting approaches to compute aerodynamic derivatives.

Aerodynamic derivatives are determined at only some fixed values of amplitudes of oscillation considering them amplitude independent. However, some studies (Scanlan, 1997a) have found the aerodynamic derivatives to be slightly amplitude dependent. Table 7 presents some examples of forced vibration analyses on different structures and the corresponding displacement amplitudes used. Walther (1994) also suggested the amplitudes of the vertical displacement h_o and rotational displacements α_o for forced oscillation simulations:

$$(11a) \quad h_o < \frac{1}{\omega} \sqrt{\frac{2\nu}{\Delta t}} = \frac{B}{2\pi} \left(\frac{v_r}{U_\infty} \right) \sqrt{\frac{2\nu}{\Delta t}},$$

$$(11b) \quad \alpha_o < \frac{1}{\omega} \sqrt{\frac{8\nu}{\Delta t}} = \frac{B}{2\pi} \left(\frac{v_r}{U_\infty} \right) \sqrt{\frac{8\nu}{\Delta t}}$$

where ω is the oscillation frequency, ν is the kinematic viscosity and Δt is the time step, respectively.

The aerodynamic derivatives are also influenced by the wind angle of attack and usually determined for several angles (Jurado *et al.*, 2008; Mannini and Bartoli, 2008). Zhu *et al.* (2002, 2006); Huang *et al.* (2012) found the notable dependence of the flutter limit of a deck section under wind inclination and yaw angle.

2.5.2. Kernel Identification Procedures

The identification of kernel functions describing unsteady aerodynamics, linear or nonlinear, is one of the major tasks when a time-domain analysis is performed. Commonly, these are the indicial and impulse functions described previously. The indicial functions can be obtained directly based on experiments (Yoshimura and Nakamura, 1979; Caracoglia and Jones, 2003a) or by a CFD approach (Brar *et al.*, 1996; Turbelin and Gibert, 1996; Miranda *et al.*, 2013; Farsani *et al.*, 2014); however, these methods require additional refinement to be used as a simple identification scheme (Wu and Kareem, 2014b). Commonly, for bluff bodies they are most often obtained by performing the Fourier transform of the exponential kernel function and equating to the flutter derivatives (Scanlan *et al.*, 1974; Scanlan, 1984; Caracoglia and Jones, 2003b,a). The impulse function can also be related to the aerodynamic derivatives using the same analogy (Bucher and Lin, 1988) as shown in Table 8 for the lift force due to vertical displacement and rotation. The rest of the terms can be obtained accordingly. The indicial function for a flat plate was approximated by a set of exponential functions through operational calculus (Jones, 1940), commonly known as Wagner-like functions. Generally, in both, impulse and indicial functions, the unsteady terms are approximated using exponential functions which are later on convoluted, while the quasi-steady values are obtained from the linearly dependent terms of the velocity and displacements. The coefficients of the exponential kernels in the indicial function approach for different types of streamlined and bluff bridge decks are obtained by using rational approximation by applying the nonlinear least-squares method (Scanlan, 1984; Caracoglia and Jones, 2003b,a; Borri and Höffer, 2000; Costa and Borri, 2006; Costa, 2007), while for the impulse function a bi-level optimisation scheme utilising the linear least-squares and Nelder-Mead algorithm is used as described by Tiffany and Adams-Jr. (1984, 1988).

An in-depth description of the identification procedure for the indicial functions was done recently by Zhang *et al.* (2011), with the extension of the overshooting phenomenon and its effect on the transient response for Aizhai bridge. Wilde *et al.* (1996) compared the different formulations of the impulse functions for bridge decks, previously used in aeronautics (e.g. Roger (1977); Karpel (1981)). Although the number of lag-terms describing the unsteady behaviour were different, there was no significant difference in the results. In bridge aerodynamics, Guo and Ge (2012); Liu and Ge (2012) introduced new formulation in the state-space. On the experimental side, Chowdhury and Sarkar (2005) proposed a new approach to directly identify the exponential coefficients based on wind tunnel testing. A comparative analysis between the approximation of indicial functions, impulse functions and their counterpart in the frequency domain has been done by Øiseth *et al.* (2011) using a statistical approach on over twenty time history analyses, resulting in of maximum 3% deviation of the flutter limit w.r.t. frequency domain proving no apparent superiority between the approximations. Recently, the rheological models have been used as kernels to approximate the linear (Diana *et al.*, 2013) and the nonlinear (Diana *et al.*, 2010) unsteady aerodynamic forces. The linear part has been obtained conventionally from the flutter derivatives by fitting several independent mechanical oscillators up to 2nd order using constrained optimisation, whilst the nonlinear by approximating the aerodynamic hysteretic behaviour. Wang and Liao (2012) used nonlinear harmonic oscillators for the approximation of the hysteresis loop representing the nonlinear aerodynamic force with higher-order unsteady effects, yielding in a rather simpler formulation. Following the same concept, Wu and Kareem (2013) utilised the ANN to model the aerodynamic nonlinear behaviour of bluff bodies. The limitation of these models is the difficulties of physical interpretation of the parameters. For the approximation of the nonlinear convolution kernels in the Volterra series, a sparse third-order scheme was identified as sufficient in the case of the Tacoma Narrows Bridge by Wu and Kareem (2014c), that would significantly reduce the computational time which presents a major setback in the Volterra series. This was followed by a framework for kernel identification based on a CFD approach utilising unit-step function by the same authors (Wu and Kareem, 2015). The comparative analysis of on the first and higher order kernels and their amplitude dependence showed the mathematical superiority of this model to simulate nonlinear bridge aerodynamics.

3. Flutter Limit Prediction

3.1. Fully-analytical - Theodorsen Theory

The analytical formulation of aeroelastic instability problems initially came from the aeronautical industry especially for the solution of flutter problem of aeroplane wings. Theodorsen (1935) gave an analytical solution based on potential flow theory for the motion-induced aerodynamic forces on a flat plate. A comparison of experimental data with the theoretical solution was made by Theodorsen and Garrick (1940). The phenomenon of flutter is characterised by coupling of oscillatory heave and pitch harmonic motion when the frequencies of these two modes coincide. The analytical flutter analysis requires simultaneous solution of the generalised equations of motion of a 2DOF system as a complex eigenvalue problem. Only the first fundamental heave mode and the first fundamental torsional mode are often considered for which a simplified 2D flutter analysis can be performed. This simplification assumes that only these selected modes participate in the flutter analysis which may lead to overestimation of the flutter limit. This approach is independent of the shape of the body but on the other hand, it neglects the effects originating from the simplification to the flat plate, therefore, this analysis is only valid for a flat plate or an airfoil. The solution of eigenvalue problem using the Theodorsen equation of motion for heave and pitch can be found in (Morgenthal, 2002; Abbas and Morgenthal, 2012a,b; Abbas, 2011, 2016).

3.2. Empirical Approach

Bleich (1948) introduced an empirical formula using Theodorsen's flutter theory for a flat plate to calculate critical flutter limit. Selberg (1961) also developed a closed-form solution by introducing the shape ratio to apply for various types of bridge cross-sections. Later Klöppel and Thiele (1967) gave a formula which is suitable for hand calculations by making use of the empirical diagrams (Svensson, 2013).

The theory of thin airfoils helps to explain the aeroelastic phenomena of torsional divergence. The wind velocity, called the divergence velocity U_d , corresponds to the occurrence of a static stability problem, as even the smallest rotation of the airfoil from the neutral position will imply displacements without limits. The divergence wind speed can be calculated by using the aerodynamic moment coefficient C_M of the deck for varying wind angles of attack θ as follows:

$$(12) \quad U_d = \sqrt{\frac{2K_\alpha}{\rho B^2 C'_M}} = B\omega_\alpha \sqrt{\frac{2I}{\rho B^4 C'_M}},$$

where K_α is the torsional stiffness of the deck. It should be noted that the divergence velocity from the above equation is only valid when the aerodynamic moment coefficient is linear for the angles of attack of interest.

It is usual that the flutter limit U_{cr} is lower than the divergence wind speed U_d for cable-supported bridges which makes static instability of less interest than flutter (Gimsing and Georgakis, 2011). Generally, the torsional frequency f_α is larger than the vertical frequency f_h in the case of cable-supported bridges with two cable planes; however, the torsional frequency may decrease with increasing wind speed due to the equivalent reduction of the torsional stiffness under the action of the aerostatic pressure. Frandsen (1966) showed that this could lead to the flutter limit for the airfoil as:

$$(13) \quad U_{cr} = U_d \sqrt{1 - \left(\frac{\omega_h}{\omega_\alpha}\right)^2}.$$

It is clear from the above equation that the flutter limit is smaller than the divergence velocity. It also appears that flutter will happen at lower wind velocity if the vertical and the torsional frequencies are close. Therefore, it is generally desired to keep the frequency ratio f_α/f_h more than two in larger cable-supported bridges (Gimsing and Georgakis, 2011). The above expression agrees reasonably well with the semi-empirical expression by Selberg (1961) who gave the following expression which is based on a theoretical thin airfoil for the

flutter limit (Tamura and Kareem, 2013):

$$(14) \quad U_{cr} = 3.7Bf_\alpha \sqrt{\frac{m\sqrt{I/m}}{\rho B^3} \left[1 - \left(\frac{\omega_h}{\omega_\alpha} \right)^2 \right]}.$$

Selberg's expression is found in slightly different form in (Gimsing and Georgakis, 2011; Holmes, 2015; Strømmen, 2006; Bartoli and Mannini, 2008). Selberg's formula may be used to provide a first estimate of the flutter stability limit for a flat plate where frequency ratio $f_\alpha/f_h > 1.5$ (Strømmen, 2006).

Rocard reported very similar empirical formula (Frandsen, 1966; Bartoli and Mannini, 2008) with the notation adopted in the present paper is as follows:

$$(15) \quad U_{cr} = 6.282f_\alpha B \sqrt{\left[1 - \frac{1}{\gamma_\omega^2} \right] \frac{r_\alpha^2 \mu}{1 + 8r_\alpha^2}},$$

where

$$(16a) \quad \mu = \pi \rho B^2 / 4m; \quad r_\alpha = \sqrt{\frac{I}{mB^2}}; \quad \gamma_\omega = \frac{\omega_\alpha}{\omega_h}.$$

Matsumoto *et al.* (2008a) also gave a simplified formula to calculate flutter limit for a flat plate using relationships between mass m and mass moment of inertia I as $I = mb^2/3$ which is very similar to Selberg's form as follows:

$$(17) \quad U_{cr} = 3.81Bf_\alpha \sqrt{\frac{\sqrt{Im}}{\rho B^3} \left[1 - \left(\frac{\omega_h}{\omega_\alpha} \right)^2 \right]}.$$

Another simplified formula which is well known for its simplicity and convenience was given by (Put, 1976) to calculate critical flutter limit as follows:

$$(18) \quad U_{cr} = \eta \left[1 + \left(\frac{\omega_\alpha}{\omega_h} - 0.5 \right) \sqrt{\frac{r}{b} 0.72\mu} \right] \omega_h b,$$

where $r = \sqrt{I/m}$ is the radius of gyration and η is an empirical form factor giving the difference of the critical wind speed of a certain profile w.r.t. flat plate prediction (1 for a flat plate). The above equation can further be simplified (Fu and Wang, 2014) as follows:

$$(19) \quad U_{cr} = \left[2.5 \sqrt{\frac{r}{b} \mu} \right] 2bf_\alpha.$$

A wide streamlined deck of a bridge shows usually a critical velocity close theoretical flat plate flutter limit, whereas bluff girders, trusses and deep deck sections show a larger difference between the theoretical flutter limit and that measured in the wind tunnel. Usually, this difference is represented with the help of a ratio of measured flutter limit to flat plate prediction β_f which is about 90% for a streamlined box girder and can be up to 40% for a bluff box sections (Gimsing and Georgakis, 2011). It would appear to be unwise to rely on a prediction based on an empirical formula alone, therefore, more detailed analyses are required for important structures and at the final design stage.

The Eurocode (2004) requires the avoidance of divergence and flutter where Eq (12) is considered to calculate the critical wind velocity for divergence. The assessment of the susceptibility of a structure is provided in terms of simple structural criteria. The structure needs to be checked for three criteria: the aspect ratio $D/B < 0.25$, the torsional axis normal to wind direction with the centre of torsion at least $B/4$

downwind of the windward edge of the section and the frequency ratio $f_\alpha/f_h < 2$. It recommends specialist advice in case these criteria are met.

3.3. Semi-analytical/ Derivative-based Analysis

3.3.1. Solution in Frequency Domain

The flutter analysis is traditionally conducted in the frequency domain as the aerodynamic derivatives are functions of reduced speed which requires an iterative solution to determine the flutter limit. Commonly, the iterative solution is performed by complex eigenvalue analysis. There are two approaches which can be followed to perform these calculations (Wenzel, 2008): first is the direct method where the unsteady aerodynamic forces are applied to the finite element model, either in the frequency domain or in the time domain, the second is mode superposition method where the structural response is separately considered in various vibration modes and to assemble them.

Flutter analysis is preferred to be performed in the frequency domain for its computational efficiency because aerodynamic derivatives in the motion-induced forces are functions of reduced frequency (Xu, 2013). There are two main analytical approaches for the solution of flutter problems: the Complex Eigenvalue Analysis (CEVA) (Simiu and Scanlan, 1996) and the Step-by-step Analysis (SBSA) (Matsumoto, 1996). The CEVA has been used for a long time for flutter problems. In the CEVA, the main challenge is related to the investigation of the system frequency and damping ratio relationship with the wind speed, whereas the SBSA is very useful for such limitations. Additionally, it describes the role of flutter derivatives on the critical condition and on flutter stabilisation (Hoa, 2004).

The motion-induced aerodynamic forces depend on the response frequency of the deck, which is determined in the analysis of complex nonlinear eigenvalue problem and then the critical flutter limit is calculated (Jurado and Hernández, 2004; Hernández *et al.*, 2008). The iterative solution is necessary because the aerodynamic derivatives depend on the reduced speed ($v_r = 2\pi U/Bf$) which is based on the oscillation frequency f which needs to be known. Xu (2013) used CEVA and computed the flutter limit of the Stonecutters Bridge. He also showed the dynamic characteristics and aerodynamic derivatives of the bridge deck with and without traffic. Dyrbye and Hansen (1997) presented modal-form solutions where the real and imaginary parts with mode shape similarity factors are given. Dung *et al.* (1998) traced the system complex mode with the increasing wind speed which helps to describe the dynamic behaviour before flutter.

Matsumoto *et al.* (1995, 1996); Matsumoto (1996) developed the SBSA and reported flutter characteristics including frequency, damping, amplitude ratio and phase difference characteristics and then compared these with the CEVA which showed reasonable agreement. Discrepancies were observed in the results obtained by the CEVA and the SBSA at higher reduced speeds beyond the flutter limit, therefore, both the damping and flutter frequency have to converge simultaneously in the iteration (Matsumoto *et al.*, 2007, 2010). Later modifications showed perfect agreement (Matsumoto *et al.*, 2007, 2010). In other studies (Matsumoto *et al.*, 1998, 1999, 2006b,a, 2008b, 2009), the complex branch switch characteristics of coupled flutter instability are focused. This method was applied to highlight the characteristics of different bluff sections (Matsumoto *et al.*, 2002, 2008c). Yang *et al.* (2006, 2009, 2007); Ge and Xiang (2008a) also proposed, based on the concept of SBSA analysis, a 2D Coupled Flutter Analysis (CFA) for 3DOF to investigate the mechanism of flutter with the focus on the aerodynamic damping, frequency and flutter modality. Chen (2007) presented closed-form formulations for estimating the modal frequencies, modal damping ratios and coupled motions of a 2D aeroelastic bridge with variation in the wind speed.

The two branches in coupled flutter are known as heaving branch (HB) and torsional branch (TB). In the SBSA, the flutter frequency in the HB and the TB are converged by iterative calculation. Based on several fundamental sections and flow-structure interaction, Matsumoto also classified flutter generation mechanism into detailed branches (Matsumoto *et al.*, 1999, 2002, 2003, 2007, 2008a,b,c) :

- Low-speed torsional flutter,
- High-speed torsional flutter,
- Heaving-branch coupled flutter,

- Torsional-branch coupled flutter and
- Heaving-torsional coupled flutter.

3.3.2. Solution in Time Domain

The frequency-domain approach for determining the flutter limit is limited to considering only a linear system under the action of the stationary wind without taking into account aerodynamic or structural nonlinearities. Therefore, a major part of recent research focuses on the time domain analysis, where it is possible to include these nonlinearities. One of the earliest implementations was by [Beliveau et al. \(1977\)](#) utilising the indicial function for time and frequency domain. Initially, the buffeting analysis was usually conducted in the time domain using the quasi-steady approach ([Diana et al., 1993](#); [Kovacs et al., 1992](#); [Miyata et al., 1995b,a](#)), with state-space model for impulse functions ([Xiang et al., 1995](#)). [Boonyapinyo et al. \(1999\)](#); [Chen et al. \(2000a,b, 2001\)](#); [Chen and Kareem \(2001a\)](#); [Thang et al. \(2008\)](#); [Øiseth et al. \(2011, 2012\)](#); [Guo and Ge \(2012\)](#) presented a time domain buffeting and flutter analysis based on the state-space using rational approximation of the impulse functions for full and reduced models in modal coordinates.

In state-space formulation, the convolution is described by a linear superposition of additional aerodynamic states representing the fluid memories. The integration is then carried out using the first order time integration schemes such as Runge-Kutta or Euler. The flutter limit is then compared to conventional frequency analysis. [Chen et al. \(2000b\)](#) presented a full-model where additional iterations are required for each step in the Newmark-beta algorithm. From the solution of second order differential equations, [Lazzari et al. \(2004\)](#) investigated the convolution integrals over the impulse functions and proposed a scheme for reduced integration for a finite time for the convolution. Both schemes proved to be successful in predicting the flutter limit for a cross-sectional model. [Borri et al. \(2002\)](#); [Costa and Borri \(2006\)](#); [Costa \(2007\)](#); [Salvatori and Borri \(2007\)](#); [Zhang et al. \(2003a\)](#); [Zhang and Chen \(2010\)](#); [Kurniawan \(2013\)](#) determined the flutter limit based on an approximation of the indicial functions for a full-model by integrating the convolution integral each time-step. To reduce the computational cost, [Borri and Höffer \(2000\)](#) presented a time window for the finite convolution of the indicial function, suggesting that only recent time has an impact on the rise of the self-excited forces.

The quasi-steady theory was applied by [Salvatori and Spinelli \(2007\)](#); [Borri and Costa \(2004\)](#); [Øiseth et al. \(2010\)](#) in the determination of the flutter limit, utilising its features to capture LCO. [Chen and Kareem \(2001b, 2000, 2003a\)](#) propose a model considering the aerodynamic nonlinearities by utilising the quasi-steady theory for high reduced velocities capturing the nonlinear part and the impulse-function based model for the low reduced velocities for the linear unsteady effects. In the case of laminar flow, this constitutes a linearisation about the nonlinear aerostatic angle of incidence. A similar method referred to as the band-superposition scheme was used by [Diana et al. \(2002\)](#) splitting the wind frequency into several bands; however, there was no clear link between the bands and it was not used in order to determine a flutter wind velocity. In [Diana et al. \(2013\)](#), the band-superposition model was extended by using the corrected quasi-steady model for the lower frequency part and the linear unsteady for the high-frequency components. The most recent models ([Wu and Kareem, 2011](#); [Diana et al., 2010, 2013](#); [Qi and Hai-li, 2012](#)) attempted to determine the flutter limit including the aerodynamic nonlinearity and the nonlinear memory terms based on the hysteretic behaviour. [Wu and Kareem \(2013\)](#) gave a comparison between the models and methods for computing the flutter limit, discussing the influence of the aerodynamic nonlinearity and linear and nonlinear fluid memory. Further comparisons for time-domain flutter analysis were done by [Lazzari \(2005\)](#); [Petrini et al. \(2007\)](#).

3.3.3. Simplified Approach

Some attempts have been made to simplify flutter analysis by developing relationship and mutual dependence of aerodynamic derivatives ([Matsumoto et al., 1996](#)) such as

$$\begin{aligned}
 (20a) \quad H_1^* &= kH_3^*; & H_4^* &= -kH_2^*; \\
 (20b) \quad A_1^* &= kA_3^*; & A_4^* &= -kA_2^*.
 \end{aligned}$$

Furthermore, there is another dependence between two pairs $(A_2^* - A_3^*)$ and $(H_1^* - H_4^*)$ because of the equivalent Wagner function. This concludes that there are only two independent aerodynamic derivatives among the eight. Scanlan *et al.* (1997) analytically described the interrelation and the dependence among the aerodynamic derivatives of an airfoil and investigated experimentally obtained aerodynamic derivatives of three different bridge deck sections. It was found that this relationship clearly does not hold for bluff sections. However, Xu (2009a,b) investigated the relationship between aerodynamic derivatives of slender bridges and based on his previously proposed semi-analytical aerodynamic derivatives of flexible structure, showed that certain relationships exist between these derivatives as follows:

$$\begin{aligned}
 (21a) \quad H_1^* &= -4A_1^*; & H_2^* &= -4A_2^* - \frac{\pi}{k}; \\
 (21b) \quad H_3^* &= -4A_3^* + \frac{\pi}{16}; & H_4^* &= -4A_4^* + \frac{\pi}{2}; \\
 (21c) \quad H_1^* &= kH_3^* - \frac{\pi G}{2}; & H_2^* &= -\frac{H_4^*}{k} - \frac{\pi F}{2k}; \\
 (21d) \quad A_1^* &= kA_3^* - \frac{\pi k}{64} + \frac{\pi G}{8}; & A_4^* &= -kA_2^* - \frac{\pi}{8}(1 - F).
 \end{aligned}$$

It also shows that semi-analytical aerodynamic derivatives are applicable to bridges with a streamlined cross-section. Additionally, Tubino (2005) provides inter-relationship among aerodynamic derivatives using generalised quasi-static theory. Al-Assaf (2006) developed an alternative analytical approach to supplement the wind tunnel testing to estimate the flutter limit of a bluff bridge section by synthesising the aerodynamic derivatives based on previous studies of a similar deck configuration. Lute *et al.* (2009) also used the aerodynamic derivative database to predict the flutter derivatives for any deck size for estimation of flutter limit. Mannini (2006) investigated flutter instability by analysing a large number of dynamic and aerodynamic data and proposed a simplified expression which uses only three or even two aerodynamic derivatives for the calculation of flutter limit (Bartoli and Mannini, 2008; Borri and Mannini, 2010; Mannini *et al.*, 2012). Lee *et al.* (2011) gave another simplified formula by applying the quasi-steady approach and using approximations in the aerodynamic derivatives for calculating 2D flutter limit. Øiseth and Sigbjørnsson (2011) presented a simplified analytical formulation, including closed-form algebraic expressions, for the prediction of the critical flutter limit. Øiseth *et al.* (2010) suggested a simplified approach to compute the flutter limit by using a new set of modified quasi-steady coefficients. Banerjee (2003) derived expressions for generalised mass, generalised stiffness and generalised aerodynamic force terms in compact explicit form. Vairo (2010) proposed an approach for the dynamic problem by considering a simple mechanical system with equivalent stiffness properties for capturing the main wind-bridge interaction mechanisms. All these simplifications generally provide a reasonable prediction for flat plate flutter but are not suitable for real bridge cross-sections.

3.3.4. Multimode Approach

The study of flutter phenomena is related to the understanding of nature of unsteady aerodynamic forces, mechanisms of mode-coupling and the methodology of multi-mode analysis (Yamada *et al.*, 2006). Scanlan (1987); Scanlan and Jones (1990) introduced single mode and 2D flutter analysis methods using modal coordinates by considering the main contribution of selected fundamental torsional mode (for torsional flutter) and coupling between pitch and heave modes (for classical flutter). The 2D coupled flutter analysis is sometimes not sufficient when the contribution of higher modes is significant. Therefore, the mode-by-mode approach can be adopted (Fujino and Siringoringo, 2013b). A 2D analysis is applicable to a case where the deck is straight, with prismatic cross section and heave and pitch modes have the same form across the span of the bridge (Tamura and Kareem, 2013). Additionally, it assumes the same wind velocity across the whole deck level.

The consideration of the horizontal modes leads to a 3D flutter analysis (Tanaka *et al.*, 1992). The lateral modes may have a significant influence on the accurate prediction of the flutter limit for long-span bridges (D'Asdia and Sepe, 1998; Mishra *et al.*, 2008b,a). The horizontal movement of the deck along the direction of the wind for very flexible long-span bridges can play an important role such as in the case of the Akashi Strait Bridge where Katsuchi *et al.* (1998, 1999); Yamada *et al.* (2006) showed that by including P_i^* aerodynamic

derivatives flutter limit reduced considerably. This significant participation of lateral modes in the flutter instability has never been observed before (Hoa, 2008). Later, Matsumoto *et al.* (2010) showed that it is due to the structural coupling of the lateral modes with the torsional mode. Aerodynamic derivatives P_i^* are usually not directly measured from wind tunnel tests, therefore, quasi-steady values may be used as shown in Table 9.

The state-of-the-art flutter analysis is generally carried out in the frequency domain by using generalised coordinate transformation in modal space sometimes using the Finite Element Method (FEM). The 3D flutter analysis uses superposition of modes in which dynamic coupling between the structural modes occurs through the self-excited aerodynamic forces (Tamura and Kareem, 2013). (Nakamura and Ogaki, 1990; Namini *et al.*, 1992; Astiz, 1998; Hua and Chen, 2008; Øiseth *et al.*, 2013; Vu *et al.*, 2015). Only a few modes are susceptible to flutter instability which are necessary to consider in the analysis. The selected modes of vibration are included in the flutter analysis which is an approximation; however, it can give a good precision if sufficient modes are chosen. The 3D effect of the structure and the wind characteristics, as well as the influence of coupled modes, are important for flutter stability analysis as the change in characteristics of the dominant mode influence the critical flutter limit. Scanlan and Jones (1990); Pleif and Batista (1995); Jain *et al.* (1996b); Katsuchi *et al.* (1998); Ge *et al.* (2002) observed, in some cases, the significant effect of higher modes in the flutter limit prediction instead of considering only fundamental bending and torsional modes. It is possible to have multiple mode coupling at the same critical wind speed when more than one curve representing the real part of the eigenvalues (related to damping) simultaneously, becomes positive (Jurado and Albarracín, 2011; Jurado *et al.*, 2013). This is very similar to multiple buckling modes of a column under axial load.

Flutter analysis is often performed in the modal space for systems with multiple modes for which the development of analytical methods has been carried out by Agar (1989, 1991). The approaches in the modal space utilise generalised coordinate transform and modal superposition techniques (Hoa, 2004, 2006, 2008). Scanlan (1978a,b); Agar (1989); Jain *et al.* (1996b); Katsuchi *et al.* (1998); Gu *et al.* (1999); Ge *et al.* (2000); Vu *et al.* (2011) have utilised multi-mode approaches to study flutter response of long-span bridges. Ge and Tanaka (2000a); Ding *et al.* (2002); Hua and Chen (2008) examined full-order methods and compared the results with the two-mode and multi-mode procedures. In the multi-mode approach, selected modes are included in the analysis whereas the full-order method considered all DOF which is accurate for flutter analysis from the viewpoint of methodology; however, requires more computation time. Chen and Kareem (2003c,b, 2004, 2006a,b, 2008) provided valuable insight into the multimode coupled flutter by introducing closed-form expressions without using CEVA for estimating modal characteristics of bridge systems and 2D flutter prediction, as well as guidance on the selection of critical structural modes and understanding of the multimode coupled bridge flutter response. Zhang *et al.* (2003b) investigated the effects of each aerodynamic derivative and mode coupling on the flutter limit.

Starossek (1993, 1997, 1998a) used the FEM to model bridge decks using beam elements to predict flutter limit for a multi-degree-of-freedom (MDOF) system. The complex representation was utilised to predict the 2D torsional flutter limit (Starossek, 1994) and coupled flutter in the mode space (Starossek, 1998b) by comparing real-number and complex-number descriptions of the same mechanical phenomena. Thiesemann *et al.* (2003); Starossek *et al.* (2009); Thiesemann and Starossek (2003) used complex representation to compare the aerodynamic derivatives from the CFD simulations based on the FVM and water tunnel tests.

3.4. Fully-coupled CFD Simulations

The 2D flow simulations on bridge decks for the prediction of coupled flutter limit have been found to give promising agreement with wind tunnel experiments (Frandsen, 2004). These simulations could be used as a supplementary tool to wind tunnel testing for preliminary analysis. Usually, the simulations are performed by free or forced oscillation methods to compute aerodynamic derivatives and then to calculate the flutter limit by using semi-analytical methods in the frequency or time domains. However, these simulations do not represent the actual response of the system at or near the flutter limit because the aerodynamic derivatives are measured at some fixed amplitude with linear assumptions and are sometimes extrapolated which may lead to erroneous results.

The FSI simulations are time domain analyses performed using the interaction between flow and a rigid body, as well as its motion. The incompressible N-S equations are fully-coupled to a rigid body which is elastically suspended on springs with lumped mass. The section of the bridge is considered to move in 3DOF; heave, pitch and lateral motion. Often only the first bending mode and the first torsional mode are considered to contribute to the response for flutter analysis. The instantaneous application of the full wind speed to an initially stationary structure leads to large transient initial motions from which it is difficult to extract definitive conclusions about the stability of small oscillations. This can be avoided by using large fictitious damping only for the first few time steps and then replacing the actual damping values (Frandsen, 2004).

CFD has the advantage of simultaneous force measurement and flow visualisation at each point of the domain. Figure 10 shows flow visualisation in a CFD simulation for bluff sections in a uniform flow. Several studies (Robertson *et al.*, 2001; Morgenthal and McRobie, 2002; Nowicki and Flaga, 2011; Zhan and Fang, 2012) have been performed to show the application and effectiveness of the approach on different test cases. Some other examples of fully-coupled CFD simulations to compute flutter limits are given in Tables 4 and 5.

The contribution of higher modes is generally low in the long-span bridge flutter, therefore, the 2D approach can provide a reasonable prediction of the flutter limit; however, this contribution can be sometimes significant and can not be ignored as discussed in the previous section. The multi-slice or quasi-3D approach (Morgenthal, 2002) can then be used to consider the additional modes where each slice acts as a separate 2D flow model and the structure is coupled with the help of structural modes. However, this approach has limitations such as the aerodynamic force correlation is not established which is insignificant for flutter analysis in a uniform flow.

3.5. Wind Tunnel Aeroelastic Models

Generally, there are two types of tests conducted in wind tunnel for the aeroelastic stability against flutter (Chen and Duan, 2014).

- Section model tests with spring supports: the aerodynamic characteristics with some selected modes (2 or 3) can be studied. Usually, the model scale is kept larger than 1/100.
- Full-3D aeroelastic model: the aerodynamic characteristics of more than 3 modes can be studied at the same time to examine coupling effect of different modes.

Some details about the method have been discussed before in Section 2.4. Table 6 provides some test examples for long-span bridges. Table 10 shows flutter limits of some well-known bridges around the world.

4. Effect of Parameters and Probabilistic Approach

4.1. Effect of Parameters on Flutter Limit

4.1.1. Structural Parameters

The effects of change in the dynamic parameters of a bridge deck model on the prediction of the aerodynamic derivatives are negligible in wind tunnel experiments (Gu *et al.*, 2001). However, uncertainty in the structural parameters can affect the flutter limit prediction significantly especially the torsional frequency and the mass moment of inertia of the deck (Abbas, 2016; Abbas and Morgenthal, 2016, 2014a). It is also important to consider the frequency ratio between the torsion and the vertical bending. Bartoli *et al.* (2008) presented the idea to avoid classical flutter by inverting this ratio to have it less than one. Moreover, Zhang and Sun (2004, 2005) studied the effect of the cable sag, side span length, section depth, dead load on the flutter stability of a suspension bridge.

4.1.2. Structural Damping

In general, structural damping ratios of up-to 2% has been used in design, however, smaller damping ratios were observed from the outcome of vibration tests of suspension bridges (Unjoh and Adachi, 1998). The use of structural damping, in flutter analysis, results in a small increase in the flutter limit; however, the variability in structural damping does not have a significant effect on the flutter limit

(Khodaparast *et al.*, 2010). Jain *et al.* (1998) observed that the installation of appropriate external dampers increases the flutter stability of a long-span bridge and can be a viable option to overcome the problems of low flutter speeds.

4.1.3. Construction Stage

The long-span bridges under erection are more vulnerable to the aerodynamic instabilities than in the final state as the stiffening girder lacks torsional continuity (Zhang, 2004). This makes it important to consider potential flutter onset especially in the early stages of deck erection where by the use of eccentric ballast flutter limit can be enhanced (Larsen, 1997). The sequences of pylons to midspan and unsymmetrical erection can play some favourable role to improve flutter stability (Ge and Tanaka, 2000b; Huang and Li, 2006; Kim *et al.*, 2011). The behaviour of Stonecutters and Sutong cable-stayed bridges against flutter was tested at construction stage by Morgenthal and Yamasaki (2010, 2011). The flutter limit of a bridge under erection may decrease also by the nonlinear effects of wind-structure interactions (Zhang, 2004). Jones *et al.* (2003) studied the effect of mode coupling on flutter during a retrofit and found that flutter can happen on a relatively short-span structure due to the peculiar modal profiles that are created as part of the retrofit procedure.

4.1.4. Deck Shape

The shape of the deck section greatly influences the flutter stability of long-span bridges. Increasing the aspect ratio (B/D) of the section, generally, improves the flutter behaviour (Lin *et al.*, 2005). Matsumoto *et al.* (1996) studied the role of various aspect ratios of a rectangular section on aerodynamic derivatives and flutter stability behaviour. Different configurations of fairings (Miranda and Bartoli, 2001; Fumoto *et al.*, 2005; Permata *et al.*, 2013), fairing angle (Wang *et al.*, 2009), vertical plates (Permata *et al.*, 2013), slot in the middle (Miranda and Bartoli, 2001; Fumoto *et al.*, 2005; Yang *et al.*, 2007, 2009, 2015), double slot and the location of porous cavity/ grating and its opening ratios (Permata *et al.*, 2013; Matsumoto *et al.*, 2001) have been studied and proposed to improve the flutter behaviour of the deck section. Abbas (2016) studied the sensitivity of cross-section geometry to the flutter limit for several classes of sections using CFD simulations.

4.1.5. Atmospheric Turbulence

The effects of turbulence on the FSI have not been fully understood and are studied for very limited cases so far (Tamura and Kareem, 2013). Atmospheric turbulence is generally considered beneficial and found to raise the flutter limit (Scanlan, 1997b, 1978b; Haan, 2000; Bartoli and Righi, 2006). In other words, free stream turbulence tends to enhance the aerodynamic stability of long-span bridges. Therefore, usually it is ignored in the measurement of aerodynamic derivatives and flutter stability analysis for simplicity (Chen and Duan, 1999; Gimsing and Georgakis, 2011). Parkinson (1989) observed that the increase in large turbulence in wind flow causes a soft galloping section to become weaker and increases stability and a hard galloping section becomes soft. In some cases the flutter limit is found to increase monotonically with increasing turbulence intensity (Lin *et al.*, 2005). The main effect of an increase in the atmospheric turbulence on a bridge is not to change considerably its drag, lift, moment and aerodynamic derivatives but to excite several modes of vibration and such response will determine to what extent the bridge is prone to flutter instability. Bucher and Lin (1988); Chen and Kareem (2003a); Wu and Kareem (2013) utilised semi-analytical models to include the effect of turbulence on flutter. Caracoglia (2009) investigated the implications of the nonlinear propagation of an uncertain turbulence field on the 2D coupled flutter stability of long-span bridges. Turbulent flow can have a negative effect on the flutter stability of long-span bridges with a streamlined section (Huang and Li, 2006). Abbas and Morgenthal (2013b); Cai and Albrecht (2000) observed that large turbulence intensity may significantly reduce the flutter velocity.

4.1.6. Nonlinear Effects

Geometric and material nonlinearities can be considered in the analysis of flutter to investigate post-flutter behaviour by performing nonlinear time history analyses and are not usually required for practical applications (Chen and Duan, 1999). However, considering the effect of wind angle on the moment coefficient may result in a significant change in the aeroelastic stability of the bridge (Cheng *et al.*, 2002). Zhang *et al.* (2002); Arena and Lacarbonara (2012) considered aerodynamic and structural nonlinearities in the flutter analysis. The effect of aerodynamic nonlinearities in the flutter analysis has already been discussed in the previous sections.

4.2. Probabilistic Approach to Flutter Analysis

The probabilistic assessment of the aeroelastic stability against flutter is essential for design consideration of cable-supported long-span bridges (Ge and Xiang, 2008a). The critical design wind speed is determined such that the probability of failure due to flutter is less than or equal to a specified target safety level. Some studies have been conducted on the probabilistic assessment of flutter limit prediction from wind tunnel testing (Ostenfeld-Rosenthar *et al.*, 1992) and CFD applications (Xiang and Ge, 2002).

The Monte Carlo Simulation (MCS) is the traditional and the most commonly used method for a probabilistic analysis. Usually, the MCS requires a large number of simulations to be performed for different sets of input parameters which may need significantly large computation time. For this purpose, the Response Surface Method (RSM) can be used to decrease the size of the problem to make this process faster. The RSM represents the structural response by using a simple approximation function. To perform a numerical simulation may take from some minutes to a few days whereas running this function requires only a fraction of a second. The computation time can be greatly reduced without modifying the existing numerical code for the simulation. These RSMs have been used for the probabilistic torsional flutter (Seo and Caracoglia, 2011a,b, 2013) and coupled flutter (Cheng and Xiao, 2005) analyses considering structural parameter uncertainties. Abbas (2016) developed a framework for probabilistic assessment of the flutter prediction models based on the concepts of total uncertainty which includes parameter uncertainty as well as model uncertainty. The probabilistic assessment also consists of sensitivity analysis, uncertainty analysis, reliability analysis and design optimisation.

4.2.1. Sensitivity Analysis

The sensitivity analysis can provide essential information about the dependence of the key chosen design variables on the flutter limit of a bridge. The objective of a sensitivity analysis is to identify important parameters and to quantify their effect on the model output. Jurado and Hernández (2004); Nieto *et al.* (2011) carried out an analytical sensitivity analysis of the flutter phenomenon in long-span bridges. Sarkar *et al.* (2007, 2009); Caracoglia *et al.* (2009) studied the sensitivities related to extracting experimental aerodynamic derivatives between different laboratory environments. Abbas (2016); Abbas and Morgenthal (2016, 2014a) studied sensitivity of structural parameters and aerodynamic derivatives to flutter limit using CFD simulations.

4.2.2. Uncertainty Analysis

Commonly a flutter analysis is performed considering complete deterministic structural parameters which is known as a deterministic analysis. However, there are uncertainties in these design variables which include, geometric properties, material properties, load magnitude and distribution (Cheng *et al.*, 2003), as well as aerodynamic derivatives (Caracoglia, 2008; Seo, 2013; Mannini and Bartoli, 2013, 2015). This makes it necessary to consider these uncertainties in the analysis for a more complete understanding. Jakobsen and Tanaka (2003) discussed uncertainties in the prediction of the aeroelastic flutter response of cable-supported bridges. Argentini *et al.* (2014) used MCS to considered the effects of structural and aerodynamic uncertainties. Structural mass and stiffness parameters are shown to have a significant influence on the flutter limit especially torsional frequency and mass moment of inertia (Abbas, 2016; Abbas and Morgenthal, 2016, 2014a).

There have been several studies conducted on the flutter stability analysis considering parameter uncertainties (Cheng *et al.*, 2003; Kwon, 2010) as well as model-form and predictive uncertainty (Riley *et al.*, 2010; Riley and Grandhi, 2010, 2011; Riley, 2011; Abbas, 2016) by using Monte Carlo simulation and utilising the advantages of the RSM. Additionally, Alduse *et al.* (2015) proposed a Bayesian approach to determine the fatigue damage using bounds and exceedance probability. Khodaparast *et al.* (2010) used interval, fuzzy and probabilistic methods to propagate the structural uncertainty through an aeroelastic analysis.

4.2.3. Reliability Analysis

The lack of complete knowledge and uncertainties in the design variables suggest a probabilistic reliability analysis of a long-span bridge to estimate the probability of failure due to flutter instability for a given return period. The probability of bridge failure due to flutter is determined by establishing the limit state as a function of the resistance variable and the load variable (Ge *et al.*, 2000). Pourzeyanali and

Datta (2002) used the basic theory of reliability considering uncertainties in geometric and mechanical properties of the bridge, modelling, damping and experimentally obtained aerodynamic derivatives. Different sampling techniques are used to reduce the computational cost for this purpose such as the importance sampling updating method (Cheng *et al.*, 2005) and the line sampling (Shufang *et al.*, 2009). Some improvements have been suggested in the flutter reliability analysis by the use of a quadratically asymptotic integral method (Li *et al.*, 2012) and using generalised formulation for random eigenvalue analysis for bridge flutter (Canor *et al.*, 2015).

4.2.4. Optimisation

Long-span bridges are very important as well as expensive structures, therefore, it is necessary to safeguard them against aeroelastic instabilities while minimising the cost. Nieto *et al.* (2008) reviewed the analytical formulation of the optimisation process whereas Hernández *et al.* (2012) presented a technique that allows the optimum design of deck shape considering both structural and aeroelastic constraints to identify the optimum bridge deck shape. Kusano *et al.* (2013, 2014, 2015) used several reliability based design optimisation methods to minimise the bridge girder weight by varying the thickness of the girder plates while satisfying the structural reliability level under flutter considering uncertainties in extreme wind velocity, as well as in aerodynamic derivatives. Xu (2014) proposed an optimisation model using a diagrammatical technique to determine the critical flutter parameters.

5. Additional Considerations

5.1. Usefulness of Aerodynamic Derivatives

The aeroelastic instability of a bridge is related to the magnitudes and signs of some of the aerodynamic derivatives of the deck cross-section. Therefore, aerodynamic derivatives can be used for preliminary examination of aeroelastic instability. The terms on the right-hand-side of the Eq. (5) can be separated into aerodynamic damping terms (associated with the velocity of motion) and aerodynamic stiffness terms (associated with the displacement of motion). The effect of the change in the damping related forces is considered to be more important for aeroelastic instability. The aerodynamic derivative related to velocity-proportional forces, describes the aerodynamic damping of that DOF and the positive sign of this derivative indicates negative aerodynamic damping. The SDOF mechanism of aerodynamic instability is related to occurrence of the negative aerodynamic damping. The aerodynamic derivative H_1^* , which is the aeroelastic lift force induced as a result of heave velocity, controls the vertical flutter and the aerodynamic derivative A_2^* , which is the aeroelastic moment induced as a result of pitch velocity, governs the torsional flutter instability. The flutter phenomenon is related to the negative aerodynamic damping whereas negative aerodynamic stiffness creates divergence.

In the case of a flat plate for SDOF instability, considering pure bending motion (refer Eq. 5a), the negative values of $H_1^* < 0$ and $H_4^* < 0$ will increase the overall system damping and stiffness, respectively. If the magnitude of the negative aerodynamic damping becomes greater than the structural damping, the amplitudes of oscillations will grow and aeroelastic instability will occur which happens usually for deep sections. Similarly, considering pure torsional motion (refer Eq. 5b), where $A_2^* < 0$ and $A_3^* > 0$ the overall system damping will increase and overall system stiffness will decrease. So it is unlikely that SDOF instability will occur and more likely to happen as 2DOF flutter (Zhan *et al.*, 2013).

Table 11 shows some guidelines for assessment of aeroelastic instability by aerodynamic derivatives of the section. The H_4^* and A_3^* tend to increase vertical stiffness and reduce torsional stiffness. For a streamlined section, $-H_3^*$ and A_4^* feed energy into structural vibration whereas aerodynamic damping terms H_1^* and A_2^* extract energy. Phase difference for streamline sections is 180° and the system is less dependent on the structural damping (Dyrbye and Hansen, 1997). Coupled flutter can still occur even if A_2^* and H_1^* are positive. Trein and Shirato (2011) reviewed the roles of aerodynamic derivatives in flutter stabilisation and pointed out some conditions that the unsteady pressure characteristics should present in flutter stabilisation.

If the instability limit exceeds the design wind speed of the site at the deck height (suitably factored for ultimate limit states), then modifications are required to be made to the deck cross-section in order to avoid the occurrence of aeroelastic instability. Section

modifications can be made to avoid the positive (H_1^* , A_2^*) aerodynamic derivatives (Chen and Duan, 1999). Matsumoto (2013) studied to stabilise flutter of long-span bridge by reduction of A_2^* through suitably combining the deck section with ($A_1^* > 0$, $A_2^* < 0$) and ($A_1^* < 0$, $A_2^* > 0$).

5.2. Counter Measures to Improve Flutter Stability

The design of long-span bridges requires to eliminate the possibility of aeroelastic instability to occur entirely within the design life. The design should ensure that the critical flutter limit of the bridge under the worst condition is still higher than the reference wind speed. Long-span bridge girders can experience unstable flutter oscillations in very strong winds. The objective of flutter control is to enhance the critical flutter limit of the bridge by introducing some modifications to the structural system. These modifications can be broadly classified into three main types based on their function as follows:

- Deck shape: changing or modifying deck cross-section shape to change the aerodynamic behaviour which results in reducing aeroelastic forces and increasing the flutter limit,
- Tuned Mass Damper (TMD): introducing TMD increases the structural damping which reduces some critical structural mode response under wind action,
- Control flaps: additional wing-like components attached to the section which often require external power and change their relative position or orientation under structural vibration.

Fujino and Siringoringo (2013b,a); Fujino *et al.* (2010, 2012) review long-span bridge vibrations under wind action and also describe some solutions as countermeasures for these vibrations. Figure 11 and Table 12 show some countermeasures to improve the behaviour of sections against flutter. Non-streamlined sections with smaller aspect ratios (B/D) are susceptible to galloping. This can be improved by using aerodynamic countermeasures such as skirts and horizontal plate on the lower side of the cross section (Fujino and Siringoringo, 2013b) which in turn reduces the wind velocity separated from the lower corner of the leading edge of the cross section thus reducing the self-excitation forces. Long and slender bridges with bluff sections of large aspect ratio are prone to torsional or coupled flutter. Low torsional rigidity and aerodynamically unstable girder cross sections are the main factors responsible for torsional flutter which caused the dramatic collapse of the original Tacoma Narrows Bridge. This can be avoided by streamlining the girder using a closed box section or by using the truss-stiffened girder with increased torsional rigidity.

Flatter streamline sections are generally prone to coupled flutter which occurs as a result of large pressure differences between upper and lower surfaces of the deck. This pressure difference can be reduced using open grating or central slot to improve behaviour against flutter instability. Several other attachments can be used to improve flutter stability such as fairings, grating, wind nose, slots, flaps and spoilers. The torsional flutter stability can be improved by attaching rails to the bottom of the fairing and vertical plates to the lower flange of the girder (Ogawa *et al.*, 2002).

Ge and Xiang (2009a) studied aerodynamic stabilisation for bridges with a central stabiliser and adopting twin-box girders. It was found that the limit of span length for a traditional suspension bridge due to aerodynamic stability can be greatly increased from about 1,500m up to 5,000m by using a widely slotted deck or a narrowly slotted deck with vertical and horizontal stabilisers. Sato *et al.* (2002) confirmed that a slotted box girder was applicable for the super long-span bridge for the reason that it is excellent in terms of economical efficiency and aerodynamic stability.

The use of a TMD on a long-span bridge has been shown to be promising for reduction of wind-induced vibration response. The effectiveness of these TMDs have been extended by changing the design from a single TMD (Chen and Kareem, 2003d) to Multiple Tuned Mass Dampers (MTMD) (Lin *et al.*, 2000) as well as nonlinear TMD (Gattulli *et al.*, 2004). Körlin and Starossek (2004); Körlin *et al.* (2012) introduced the rotational mass damper, the movable eccentric mass damper and mass damper system with two eccentric

rotational actuators for active bridge deck flutter control. Using the concept of eccentric mass, [Phongkumsing *et al.* \(2001\)](#) presented a method of suppression of flutter by placing the auxiliary mass on the windward side of a bridge deck to shift the centre of gravity, and thus, reducing the aerodynamic moment acting on the deck. [Aslan and Starossek \(2008\)](#) presented a passive aeroelastic damper for long span bridges which consists of a TMD, control surfaces, and a transmission which couple the movement of the TMD with the control surfaces without the need for an external energy supply.

In some cases, the dissipative dampers working in the horizontal direction between deck and piers, resulted ineffective against flutter. This has been evaluated through a refined model of the controlled bridge with wind interaction forces ([Domaneschi and Martinelli, 2014](#)). The devices implemented in the horizontal plane do not influence the torsional stiffness of the bridge deck and therefore the flutter limit is not affected. Furthermore, the exchanged control forces between the deck and the towers, quite far from the centre span, are not able to improve the flutter stability behaviour.

The aerodynamic-based control method also has proven to be effective towards the aerodynamic stability. This method uses control surfaces to generate forces counteracting the unstable oscillations. [Graham *et al.* \(2011\)](#) used an integrated controllable trailing-edge flap and [Huynh and Thoft-Christensen \(2001, 2000\)](#) used separate control flaps to shown effective increase in the critical flutter limit of a suspension bridge. [Phan and Nguyen \(2013\)](#) focused on a mechanically controlled system using the passive flap control driven by the motions of the bridge deck. [Li *et al.* \(2015\)](#) presented an active aerodynamic control method of flutter oscillation comprising of a twin-winglet system which can improve aerodynamic stability to some extent.

5.3. Beneficial Use of Flutter

Flutter is generally considered as a destructive phenomenon but it can be turned into an environmental friendly energy source by utilising small plate-like devices as energy harvesters. Energy harvesters are typically small-scale which transform kinetic energy of wind into electrical energy to provide power to small-scale electronic devices such as wireless sensors and mobile electronic devices ([Singh *et al.*, 2012](#)). The most commonly used system is based on the LCO of flexible plate by coupling aerodynamic and nonlinear equation of motion of the plate. [Tang *et al.* \(2003\)](#) show experimental investigation to study flutter and LCO of plates. [Peng and Chen \(2012\)](#) present some background and state-of-the-art developments in this field. Energy harvesting is an active research field and is growing as the demand for renewable energy sources continues to increase ([Park *et al.*, 2014](#)). In addition, [Matsumoto \(2012\)](#) studied a mechanism of generating propulsion force by flapping a plate for a small ship.

6. Concluding Remarks of Bridge Flutter

The aim of this paper is to provide an overview of existing methods and models used to predict flutter limit by pointing out the available literature. Some important conclusions are drawn in the light of this review. Theodorsen's flat plate prediction is simple and easy to use but is only for 2D flow and flat plates or airfoils which cannot be used for bridge cross-sections. However, it serves as a starting point or reference value to compare actual flutter limits. Scanlan's model is more commonly used as it gives a more direct assessment about the nature of the aerodynamic behaviour of the cross-section.

Frequency domain methods are commonly used because of their simplicity and efficiency; however, they are only for linear structure models and do not consider nonlinearities. Therefore, the major part of the current research on the analytical models is to develop the time domain methods in order to consider the aerodynamic nonlinearities and the fading fluid memory. Simplified approaches are useful to understand the mechanism related to flutter phenomenon but cannot be trusted for the final design. CFD is still a developing field and at this point provides a useful alternative to wind tunnel tests only for preliminary design but for final designs, wind tunnel tests are preferred.

The choice of the bridge deck section is important where the aerodynamic behaviour is governing design considerations. It can save a lot of effort and resources if suitably selected in the early design stages. Changing the shape of deck is much more effective. The MTMD

777 is more desirable than flaps and control systems which require an external power supply. The probabilistic approaches provide very
778 important insight into the phenomena. The future of aeroelastic analysis for flutter prediction is the use of CFD approaches coupled with
779 semi-analytical methods.

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Figure 1. The original Tacoma Narrows Bridge collapse, USA (source: <http://www.lib.washington.edu>)

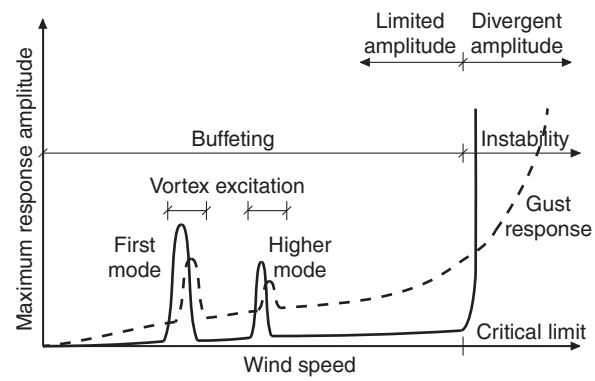


Figure 2. Schematic of structural response against wind speed, (—) uniform flow (---) turbulent flow.

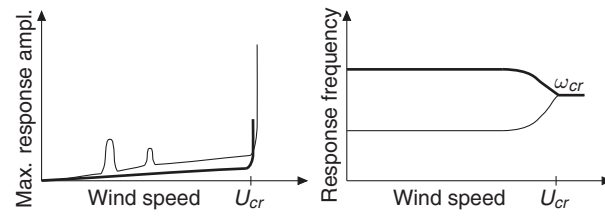


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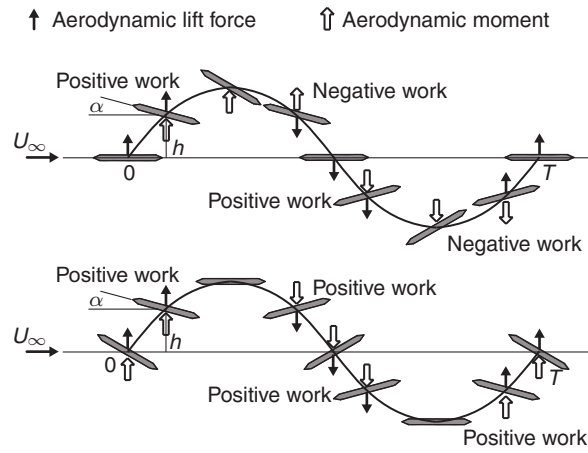


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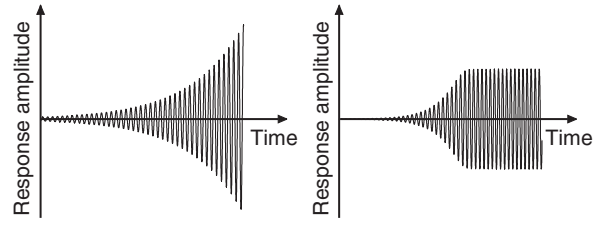


Figure 5. Schematic of (left) flutter instability and (right) limit cycle oscillation.

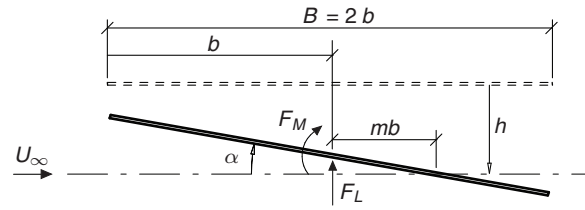


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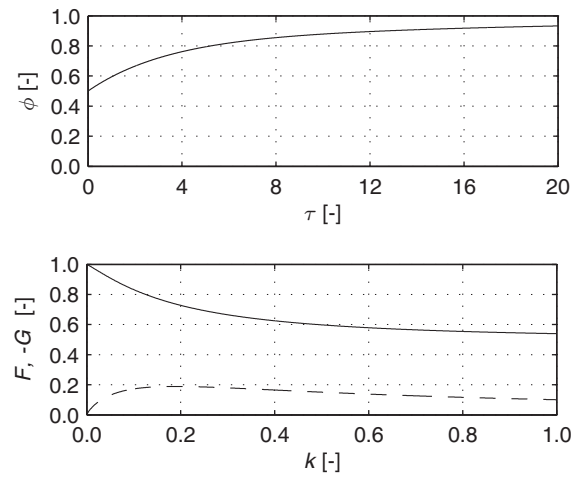


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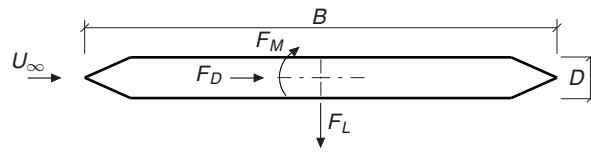


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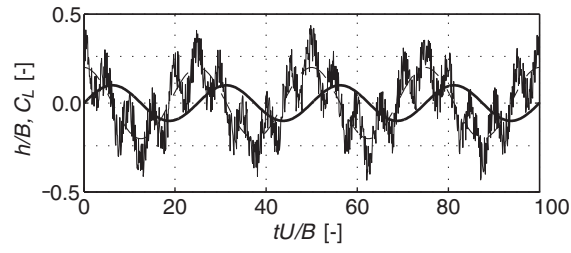


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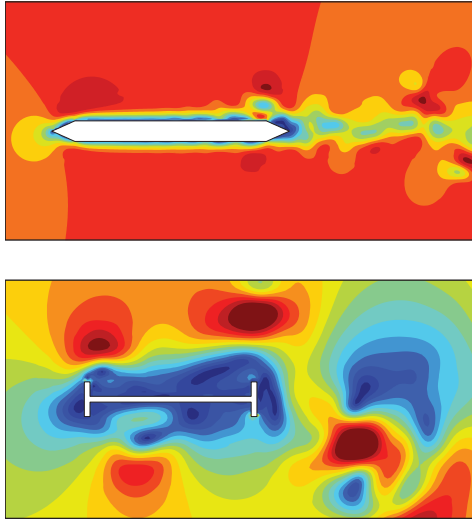


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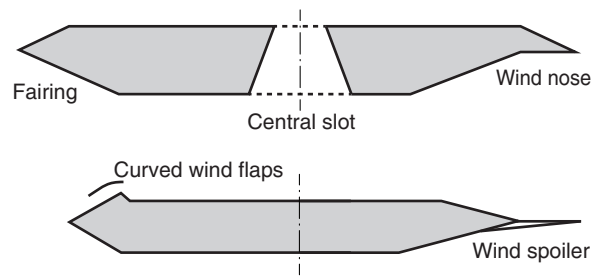


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Bridge name	Country	Year built	Main span [m]	Deck width [m]	Mass [10 ³ kg/m]	Mass moment of inertia [10 ³ kgm ² /m]	Bending frequency [Hz]	Torsional frequency [Hz]
Suspension bridge								
Messina	Italy	Planned	3300	60.4	55.00	28000	0.061	0.080
Akashi-Kaikyo	Japan	1998	1991	35.5	43.79	9826	0.064	0.150
Zhejiang Xihoumen	China	2008	1650	36.3	-	-	0.100	0.232
Great Belt (Storebælt)	Denmark	1998	1624	31.0	22.70	2470	0.099	0.272
Little Belt (Lillebælt)	Denmark	1970	600	33.0	11.66	1018	0.156	0.500
Tacoma Narrows	USA	1940	854	12.0	4.25	178	0.130	0.200
Hålogaland	Norway	2017	1145	18.6	11.94	355	0.143	0.441
Bosporus	Turkey	1973	1074	28.0	13.55	1352	0.162	0.371
Runyang	China	2005	1490	-	-	-	0.124	0.225
Tsing Ma	Hong Kong	1997	1377	-	-	-	-	-
Humen	China	1997	888	-	-	-	0.112	0.426
Cable-stayed bridge								
Russky	Russia	2012	1104	-	-	-	-	-
Sutong	China	2008	1088	41.0	-	-	0.171	0.521
Stonecutters	Hong Kong	2009	1018	53.3	32.60	297	0.172	0.324
Indiano	Italy	1978	189	22.4	17.40	545	0.573	1.179
Guama	Brazil	-	320	14.2	22.51	567	0.331	0.649
Tsurumi	Japan	1994	510	38.0	32.22	2880	0.204	0.486
Normandy	France	1995	856	23.8	13.70	633	0.222	0.500
Footbridge								
Siena	Italy	2006	59	3.3	1.42	0.810	1.735	4.839
Turin	Italy	2003	156	6.9	3.35	13.502	0.510	0.590
Kehl-Straßburg	Germany-France	-	183	6.6	-	-	-	0.650

Table 1. Some well-known long-span cable-supported bridges with their structural properties (Ostenfeld *et al.*, 1970; Larsen, 1998; Kvamstad, 2011; Selvam and Govindaswamy, 2001; Larsen, 2006; Bartoli and Mannini, 2008; Ge and Xiang, 2008b; Haifan, 2011; Morgenthal *et al.*, 2005).

Scanlan	H_4^*	H_1^*	A_4^*	A_1^*	H_3^*	H_2^*	A_3^*	A_2^*
Starossek	$\frac{\pi}{2} c'_{hh}$	$\frac{\pi}{2} c''_{hh}$	$\frac{\pi}{4} c'_{\alpha h}$	$\frac{\pi}{4} c''_{\alpha h}$	$\frac{\pi}{4} c'_{h\alpha}$	$\frac{\pi}{4} c''_{h\alpha}$	$\frac{\pi}{8} c'_{\alpha\alpha}$	$\frac{\pi}{8} c''_{\alpha\alpha}$
Küssner	$\frac{-v_r^2}{2\pi} k_1^a$	$\frac{-v_r^2}{2\pi} k_2^a$	$\frac{-v_r^2}{4\pi} m_1^a$	$\frac{-v_r^2}{4\pi} m_2^a$	$\frac{-v_r^2}{4\pi} k_1^b$	$\frac{-v_r^2}{4\pi} k_2^b$	$\frac{-v_r^2}{8\pi} m_1^b$	$\frac{-v_r^2}{8\pi} m_2^b$
Zasso	$\frac{\pi}{2} h_4^*$	$\frac{-v_r}{2\pi} h_1^*$	$\frac{\pi}{2} a_4^*$	$\frac{-v_r}{2\pi} a_1^*$	$\frac{v_r^2}{4\pi^2} h_3^*$	$\frac{-v_r}{2\pi} h_2^*$	$\frac{v_r^2}{4\pi^2} a_3^*$	$\frac{-v_r}{2\pi} a_2^*$
Jensen	$-Y'_r$	$Y'_v \frac{1}{K}$	$-N'_r$	$N'_v \frac{1}{K}$	$-(Y'_r + Y'_v \frac{1}{K^2})$	$(Y'_r - Y'_v) \frac{1}{K}$	$-(N'_r + N'_v \frac{1}{K^2})$	$(N'_r - N'_v) \frac{1}{K}$

Table 2. Conversion between the notations of the aerodynamic derivatives by Scanlan, Starossek, Küssner, Zasso and Jensen.

Scanlan	H_1^*	H_2^*	H_3^*	H_4^*	A_1^*	A_2^*	A_3^*	A_4^*
Quasi-steady	$-(C'_L + C_D) \frac{v_r}{2\pi}$	$-(C'_L + C_D) m_L \frac{v_r}{2\pi}$	$C'_L \frac{v_r^2}{(2\pi)^2}$	$-2C_L \frac{v_r}{2\pi}$	$-C'_M \frac{v_r}{2\pi}$	$C' M m_M \frac{v_r}{2\pi}$	$C'_L \frac{v_r^2}{(2\pi)^2}$	$-2C_M \frac{v_r}{2\pi}$

Table 3. Conversion between the notations of the aerodynamic derivatives by Scanlan and corrected quasi-steady theory.

Structure	Phenomena	Reference
NACA-0012	D, R	Maïytre <i>et al.</i> (2003)
Rectangular, $B/D = 8$	D	Thiesemann <i>et al.</i> (2003)
Great Belt, 200mm	D	Thiesemann and Starossek (2003)
Millau Viaduct, 200mm	D	Thiesemann and Starossek (2003)
Tacoma Narrows, 200mm	D	Thiesemann and Starossek (2003)
Slotted rectangular, 450mm	P, D	Shirai and Ueda (2003)
Slotted flat box, 370mm	P, D	Shirai and Ueda (2003)
Great Belt	C, D	Vairo (2003)
Normandy	C, D	Vairo (2003)
π -sec with $B/D = 6$	A_2^*	Diogo <i>et al.</i> (2005)
Great Belt	D	Stærdahl <i>et al.</i> (2007)
Hume, 445×33.4mm	D, F	Xin and Ou (2007)
Slotted box girder	C	Watanabe and Fumoto (2008)
Great Belt	F	Kalesar and Ali (2008)
Tacoma	F	Kalesar and Ali (2008)
Box girder, $B/D = 11.6$	C, D, F	Sarwar <i>et al.</i> (2008)
Rectangular, $B/D = 4$	C, D	Sun <i>et al.</i> (2008)
Rectangular, $B/D = 4$	C, D	Sun <i>et al.</i> (2009)
Flat plate	D	Szabó and Györgyi (2009a)
Bridge section	D	Szabó and Györgyi (2009a)
Incheon, 200×30mm	D	Szabó and Györgyi (2009b)
Nine bridge decks, 200mm	D, R	Starossek <i>et al.</i> (2009)
Five bridge sections	D	Huang <i>et al.</i> (2009)
Hume, 445×33.4mm	D, F	Xin <i>et al.</i> (2010)
Circular cylinder	C	Brusiani <i>et al.</i> (2010)
Great Belt, 375×54.7mm	D	Brusiani <i>et al.</i> (2010)
NACA0012	C, D	Bai <i>et al.</i> (2010)
U-shape, G1, G2	C, D	Bai <i>et al.</i> (2010)
Four generic sections	C, D, L	Szabó and Györgyi (2011)
Thin flat plate, 700×3.5mm	D	Haug and Liao (2011)
2nd Nanjing Yangtze, 772×63mm	D	Haug and Liao (2011)
Bridge deck, 369×60mm	P, C, D	Šarkiċ <i>et al.</i> (2011)
Bridge deck, 366×66mm	P, C, D	Šarkiċ <i>et al.</i> (2012)
Rectangular, 300×60mm	F	Liua <i>et al.</i> (2012)
Streamlined, 404×60mm	F	Liua <i>et al.</i> (2012)
Great Belt, 31×4.34m	D, F	Zhan <i>et al.</i> (2013)
Circular cylinder	C	Brusiani <i>et al.</i> (2013)
Great Belt, 375×54.7mm	D	Brusiani <i>et al.</i> (2010)
R-5, 8, 10, 12.5, 15, 20	C, D, F	Miranda <i>et al.</i> (2014)
Flat plate, 1000×10mm	P, D	Ketong and Aiping (2014)
Great Belt, 31×4.4m	P, D	Ketong and Aiping (2014)
Bridge deck, 366×66mm	P, C, D	Šarkiċ (2014)
Bridge deck, 366×66mm	P, C, D	Šarkiċ <i>et al.</i> (2015)
Rectangular cylinder, $B/D = 4$	C, D	Nieto <i>et al.</i> (2015a)
Rectangular cylinder, $B/D = 4.9$	C, D	Nieto <i>et al.</i> (2015b)
G1 section	C, D	Nieto <i>et al.</i> (2015b)
Six deck sections, 366×66mm	R, C, D, F	Patruno (2015)

Table 4. Different CFD approaches based on FVM and FEM used to study aerodynamic phenomena (F: Flutter limit, D: Aerodynamic derivative, C: Static wind coefficients, P: Pressure, R: Reynolds number dependence).

Structure	Phenomena	Reference
Great Belt, 31000×4337mm	P, C, D, F	Walther (1994)
Great Belt	D, F	Larsen and Walther (1996)
Flat plate, $B/D = 200$	D	Walther and Larsen (1997)
Tacoma Narrows, 12×2.4m	C, D	Larsen and Walther (1997a,b)
Great Belt, 31×4.4	C, D	Larsen and Walther (1997a,b)
Approach span, 25.8×7m	C, D	Larsen and Walther (1997a,b)
Gibraltar APP, 65×2.5m	C, D	Larsen and Walther (1997a)
Five generic sections, G1-G5	C, D	Larsen and Walther (1998)
Five generic sections, G1-G5	C, D	Larsen (1998)
Great Belt, 31×4m	D, F	Morgenthal (2000)
Flat plate, $B/D = 100$	D, F	Morgenthal (2002)
Millau Viaduct, 388×60.2mm	D, F	Morgenthal (2002)
Great Belt, 31×4m	C, D	Morgenthal and McRobie (2002)
H-shaped	C	Larsen and Walther (2003)
Humen and Great Belt	C	Zhou <i>et al.</i> (2003)
Nanjing and Runyang	C	Zhou <i>et al.</i> (2003)
Jinsha	D, F	Zhou <i>et al.</i> (2003)
Tacoma Narrows	C, A_2^*	Zhou <i>et al.</i> (2006)
Stonecutters	C, D, F	Janjic <i>et al.</i> (2008)
Tacoma Narrows	A_2^*	McRobie <i>et al.</i> (2013)

Table 5. Different CFD approaches based on the vortex method used to study aerodynamic phenomena (F: Flutter limit, D: Aerodynamic derivative, C: Static wind coefficients, P: Pressure, R: Reynolds number dependence).

Structure	Size $B \times D$ [mm] (B/D [-])	Response	Reference
Rectangular	200×20	Coupled-Forced	Matsumoto <i>et al.</i> (1993)
Rectangular	200×10	Coupled-Forced	Matsumoto <i>et al.</i> (1993)
Box girder	292.7×33.7	Free	Iwamoto and Fujino (1995)
Flat plate	450×20	Free	Gu <i>et al.</i> (2000)
Jiangyin, 1:70	36.9×3.0m	Free	Gu <i>et al.</i> (2000)
Streamlined	-	Free, Buffeting	Brownjohn and Jakobsen (2001)
Dual carriageway	-	Free, Buffeting	Brownjohn and Jakobsen (2001)
Plate, 2DOF, 3DOF	460×45	Free	Chen <i>et al.</i> (2002)
Streamlined box	450×32	Free	Chen <i>et al.</i> (2002)
P-shaped	452×46	Free	Chen <i>et al.</i> (2002)
NACA-0020, 3DOF	314	Free	Chowdhury and Sarkar (2003)
Chamfered plate	(25)	Forced	Wu <i>et al.</i> (2006)
Rectangle	(5,8,10,12.5,15,20)	Free	Chen <i>et al.</i> (2008)
NACA-0020	456	Forced	Neuhaus <i>et al.</i> (2009)
Tacoma Narrows	411×82	Forced	Neuhaus <i>et al.</i> (2009)
Strelasund	324×103	Forced	Neuhaus <i>et al.</i> (2009)
Model A, 1:250	123×13	Free, Forced	Piña and Caracoglia (2009)
Model B, 1:500	61.33×6.35	Free, Forced	Piña and Caracoglia (2009)
Extradosed, 1:100	295.4	Forced	Hunyadi (2009)
Flat plate	450×20	Free	Boonyapinyo and Janesupasaeree (2009)
Industrial-Ring-Road	399×35	Free	Boonyapinyo and Janesupasaeree (2009)
Industrial-Ring-Road	399×35	Free	Boonyapinyo <i>et al.</i> (2009)
Fairing-modified, 1:90	399×35	Free	Boonyapinyo <i>et al.</i> (2009)
Soffit plate modified	399×35	Free	Boonyapinyo <i>et al.</i> (2009)
Combination of two	399×35	Free	Boonyapinyo <i>et al.</i> (2009)
Single-box	450×70	Free	Bartoli <i>et al.</i> (2009)
Multiple-box girder	609×52	Free	Bartoli <i>et al.</i> (2009)
Great Belt, 1:60	31×4m	Free	Larsen <i>et al.</i> (2011)
Sutong	812	Free	Xu <i>et al.</i> (2012)
Suramadu	600	Free	Xu <i>et al.</i> (2012)
Flat plate	-	Free	Xu <i>et al.</i> (2014)
Sutong	41×4m	Free	Xu <i>et al.</i> (2014)
Stonecutters	53.3×4m	Free	Xu <i>et al.</i> (2014)
Akashi-Kaikyo	35.5×14m	Free	Xu <i>et al.</i> (2014)

Table 6. Wind tunnel tests performed for different structures to obtain aerodynamic derivatives.

Structure	Size $B \times D$ [mm] (B/D [-])	Heave amplitude h_o/B [-] (h_o [mm])	Pitch amplitude α_o [°]	Reference
Rectangular	200×20	$h_o/\alpha_o = 0.0637$		Matsumoto <i>et al.</i> (1993)
Rectangular	200×10	$h_o/\alpha_o = 0.1408$		Matsumoto <i>et al.</i> (1993)
Great Belt	31000	0.05	5	Walther (1994)
Five generic	-	0.05	3	Larsen (1998),
sections, G1-G5	-	0.05	3	Larsen and Walther (1998)
Flat plate	(100)	0.50	3	Morgenthal (2002)
Great Belt	200	0.04	5	Thiesemann and Starossek (2003)
Tacoma Narrows	200	0.04	5	Thiesemann and Starossek (2003)
Rectangular	200	0.04	5	Thiesemann and Starossek (2003)
Rectangular	200 (8)	0.04	8	Thiesemann <i>et al.</i> (2003)
Great Belt	200	0.04	8	Thiesemann <i>et al.</i> (2003)
Millau Viaduct	200	0.04	8	Thiesemann <i>et al.</i> (2003)
Tacoma Narrows	200	0.04	8	Thiesemann <i>et al.</i> (2003)
π -section	12000×2000	-	1.72	Diogo <i>et al.</i> (2005)
Thin plate	40000(100)	(100)	3	Ge and Xiang (2008c)
H-shape	11900	(100)	3	Ge and Xiang (2008c)
Great Belt	31000	(100)	3	Ge and Xiang (2008c)
PWRI	-	0.01	1	Sarkar <i>et al.</i> (2009)
ISU	300	0.05	2.8	Sarkar <i>et al.</i> (2009)
Rectangular	(2-5)	0.01-0.20	2.8	Sarkar <i>et al.</i> (2009)
Nine bridge decks	200	0.01, 0.02, 0.04	2, 5, 8	Starossek <i>et al.</i> (2009)
Model A, 1:250	123×13	(0.5-1.49)	1.77-2.35	Piña and Caracoglia (2009)
Model B, 1:500	61.33×6.35	(5-6)	3.43	Piña and Caracoglia (2009)
Thin flat plate	700×3.5	(2.5-6.12)	0.35-0.86	Huang and Liao (2011)
2nd Nanjing Yangtze	772×63	(2.5-5.59)	0.40-0.89	Huang and Liao (2011)
Great Belt East, 1:60	31000×4000	(16)	2	Larsen <i>et al.</i> (2011)
Bridge Section	369.5×60.6	(4)	1	Šarkić <i>et al.</i> (2011)
Bridge Section	366×66.6	(4)	1	Šarkić <i>et al.</i> (2012)
Bridge Section	366×66.6	(4)	1	Šarkić (2014)
Bridge Section	366×66.6	(4)	1	Šarkić <i>et al.</i> (2015)

Table 7. Amplitudes of heave and pitch used for forced vibration tests.

Indicial/Impulse	$\bar{\Phi}_{Lh}(k) + \Phi_{Lh}(0)$	$\bar{\Phi}_{L\alpha}(k) + \Phi_{Lh}(0)$	$\bar{I}_{Lh}(k)$	$\bar{I}_{L\alpha}(k)$
Scanlan	$\frac{K}{C_L} (H_1^*(K) - iH_4^*(K))$	$\frac{K^2}{C_L} (H_3^*(K) + iH_2^*(K))$	$\frac{K^2}{2} (H_4^*(K) + iH_1^*(K))$	$\frac{K^2 b}{2} (H_3^*(K) + iH_2^*(K))$

Table 8. Relations between the Fourier transform of indicial and impulse functions and aerodynamic derivatives (after Caracoglia and Jones (2003b) and Chen and Kareem (2002)).

Scanlan	P_1^*	P_2^*	P_3^*	P_5^*	H_5^*	A_5^*	H_6^*, A_6^*	P_4^*, P_6^*
Quasi-steady	$-\frac{1}{K}C_D$	$\frac{1}{2K}C_D'$	$\frac{1}{2K^2}C_D'$	$\frac{1}{2K}C_D'$	$\frac{1}{K}C_L$	$-\frac{1}{K}C_M$	0	0

Table 9. Conversion between the notations of the aerodynamic derivatives by Scanlan and Quasi-steady theory (after [Xu \(2013\)](#)).

Bridge name	Main span [m]	Girder type	Flutter limit [m/s]	Vibration countermeasures
Suspension				
Messina	3300	Triple box	75	Central slot
Akashi-Kaikyo	1991	Truss	84	Slotted deck, Stabiliser, Open Grating
Zhejiang Xihoumen	1650	Twin box	78	Slotted deck, Central slot
Great Belt (Storebælt)	1624	Single box	60	Guide vanes, spoiler
Little Belt (Lillebælt)	600	Single box	100	Guide vanes
Tacoma Narrows	854	H-shape	-	Fairings (planned)
Runyang	1490	Single box	75	Centre vertical stabiliser
Tsing Ma	1377	Single box	74	Central slot
Humen	888	Single box	88	-
Hålogaland	1145	Single box	71	-
Cable-stayed				
Russky	1104	Single box	-	-
Sutong	1088	Single box	88	Damper
Stonecutters	1018	Twin box	140	Damper
Normandy	856	Single box	78	-

Table 10. Long-span cable-supported bridges with their flutter limits and countermeasures (as of August 2015) (Fujino and Siringoringo, 2013b; Xu, 2013; Xin and Ou, 2007; Ge and Xiang, 2008a,b; Myerscough, 2013; Kreis and André, 2005; Ostfeld *et al.*, 1970; Haifan, 2011).

Name	Condition	Type of motion	Type of section
Galloping	$H_1^* > 0$	Translational	D-shape, Square section, Deep sections
Torsional flutter	$A_2^* > 0$	Rotational	H-section, Rectangle with smaller aspect ratio
Classical flutter	$H_2^* > 0, A_1^* > 0$	Coupled	Flat plate, airfoils

Table 11. Types of aerodynamic instabilities (after [Holmes \(2015\)](#); [Gimsing and Georgakis \(2011\)](#)).

Instability phenomenon	Girder type	Countermeasure
Galloping	deep box	low skirts, horizontal plates
Torsional flutter	H-section, Rectangle with smaller aspect ratio	fairings
Classical flutter	deep truss streamlined	open grating, vertical stabiliser fairing, central slots, curved wind flaps, wind nose, wind spoilers

Table 12. Typical aerodynamic countermeasure used for bridge sections against aeroelastic instability phenomena (after Fujino and Siringoringo (2013b)).