

Buffeting Analysis of Structures using Gaussian Processes with Semi-analytical Priors

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SUMMARY:

Wind-induced vibrations are commonly the leading design criterion for slender line-like structures. The paper presents a hybrid methodology for modeling the aerodynamic forces acting on bluff bodies. It comprises a hybrid model consisting of the quasi-steady model, as a semi-analytical, prior physics-based knowledge, and a Gaussian Process (GP) data-driven model to capture the model inadequacy of the former due to the quasi-steady assumption. The presented model takes non-dimensional inputs in the form of motion- and gust-induced angles of attack and outputs a probability distribution of the lift and moment coefficients. A novel training procedure based on Computational Fluid Dynamics (CFD) is devised, which accounts for concurrent forced excitation due to input motion and free-stream turbulence with prescribed spectral characteristics. The predictive capabilities of the models are demonstrated on forced vibration with large angles of attack and buffeting response at various wind speeds.

Keywords: Buffeting, Machine Learning, Gaussian Processes, Aerodynamic Nonlinearities

1. INTRODUCTION

Sharp edges and significant flow separation at high Reynolds numbers, typical in bluff cross-sections such as those found in long-span bridges or tall towers, pose considerable challenges in modeling aerodynamic forces in structural aerodynamics. Traditional approaches often rely on semi-analytical models that use either quasi-steady or linear unsteady assumptions to predict these forces. However, these models typically account for either linear fluid memory or quasi-steady nonlinearity alone. Computational Fluid Dynamics (CFD) models are also employed for predicting aeroelastic response, but they can be computationally intensive (cf. Kareem, 2020).

Recently, machine learning methods have gained attention for data-driven modeling of aerodynamic forces due to their ability to capture complex, nonlinear relationships (cf. Abbas et al., 2020). These models learn the underlying nonlinear input-output function between time-dependent angle of attack and aerodynamic forces directly from data, capturing both unsteadiness and quasi-steady nonlinearity. However, purely data-driven models may lack robustness. To address this challenge, integrating physical knowledge into data-driven models, resulting in hybrid or grey box models, can enhance robustness and efficiency.

This study introduces a hybrid modeling methodology using Gaussian Processes (GP), a probabilistic machine learning method, with a semi-analytical model as prior knowledge (cf. Fig. 1). Building on a recent GP model of motion-induced forces developed by Kavrakov et al. (2022), the methodology incorporates the non-dimensional gust-induced angle as an additional input and integrates a nonlinear quasi-steady (QS) model as prior information. This approach leverages the QS model's representation of physics-based knowledge within the system. The model is applied to conduct buffeting analysis of the Great Belt Bridge deck based on CFD training data, with potential applicability to wind tunnel data as well.

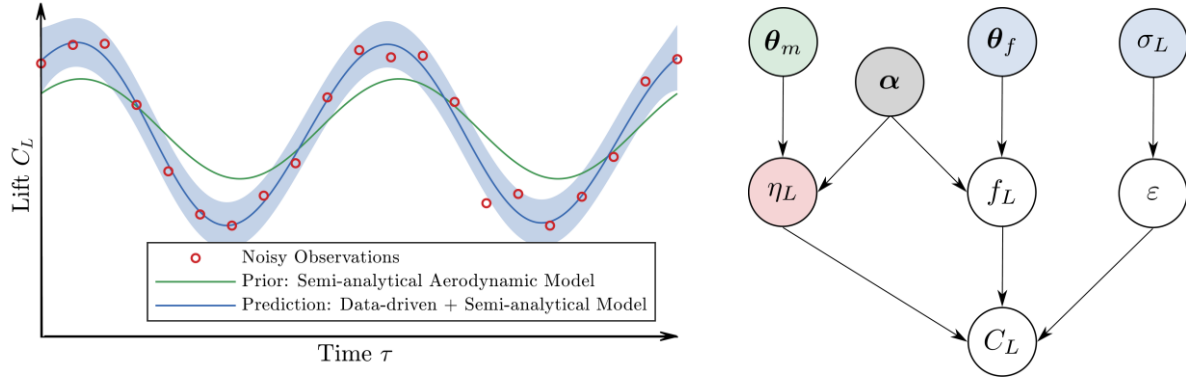


Figure 1. Concept of the presented hybrid model in time series (left); corresponding graphical model (right).

2. ENHANCED GAUSSIAN PROCESS MODEL WITH AERODYNAMIC PRIORS

Consider the wind-structure interaction system depicted in Fig. 2. The aerodynamic force model is constructed as a hybrid model, comprising semi-analytical and data-driven components, as depicted in the graphical model of system in Fig. 1 (right). For instance, the lift force coefficient C_{Li} at discrete time-step i is formulated as:

$$C_{Li} = \eta_{Li}(\alpha_{e,i}) + f_L(\alpha_i) + \varepsilon_{L,i}(t) \quad (1)$$

where $\eta_{Li} = C_L(\alpha_{e,i})$ is the QS model as an aerodynamic prior with the non-dimensional effective angle of attack $\alpha_e(t)$ as an input:

$$\alpha_e(t) = \alpha + \arctan\left(\frac{w + \dot{h} + \mu_L \dot{B}\alpha}{U + u}\right), \quad (2)$$

where μ_L is the aerodynamic center, obtained from the aerodynamic derivatives (cf. Diana and Omarini (2020)).

The model discrepancy, i.e. the physics that is intractable by the QS model due to its inherent assumptions, is modelled as a nonlinear Gaussian Process prior $f_L \sim \mathcal{GP}(0, k_L(\alpha_i, \alpha_j; \theta))$, with a covariance function $k = k(\alpha_i, \alpha_j; \theta)$ and hyperparameters θ ; the noise $\varepsilon_L = \mathcal{N}(0, \sigma_L^2)$ is assumed to be Gaussian. The non-dimensional input vector consists of motion- and gust-induced angles $\alpha = (\alpha_h, \alpha_a, \alpha_u, \alpha_w, \alpha'_h, \alpha'_a, \alpha'_u, \alpha'_w)$, $\alpha_h(t) = h'/B$, $\alpha_a(t) = \alpha$; $\alpha_u(t) = u/U$; $\alpha_w(t) = w/U$, which are lagged $f_L = f_L(\alpha_i, \dots, \alpha_{i-S})$ up to time step S . The derivative with respect to non-dimensional time $\tau = tU/B$ is denoted with a prime. We utilize the exponential covariance function with Automatic Relevance Detection (ARD) (cf. Rasmussen and Williams (2006)), which is infinitely differentiable; therefore, is particularly suitable for applications for dynamical systems. Due to the ARD property, the exponential kernel scales each dimension of the input vector independently; thereby, making a natural choice for structural dynamics applications as it can be directly related to the fluid memory (i.e. each lag term is scaled independently).

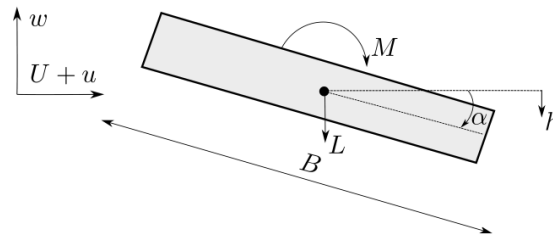


Figure 2. Wind-structure interaction.

Learning of the nonlinear function f_L , i.e. the kernel hyperparameters θ , is achieved through the minimization of the log-likelihood (cf. Rasmussen and Williams (2006)):

$$\log p(\mathbf{C}_L | \boldsymbol{\alpha}, \boldsymbol{\eta}_L; \theta) = -\frac{1}{2}(\mathbf{C}_L - \boldsymbol{\eta}_L)(\mathbf{K}_L + \sigma_L^2 \mathbf{I})^{-1}(\mathbf{C}_L - \boldsymbol{\eta}_L) - \frac{1}{2} \log |\mathbf{K}_L + \sigma_L^2 \mathbf{I}| - \frac{N_p}{2} \log 2\pi, \quad (3)$$

where $\mathbf{K}_L = k_L(\boldsymbol{\alpha}, \boldsymbol{\alpha}'; \theta)$ is the covariance matrix, constructed based on the kernel function k at input training points $\boldsymbol{\alpha}$, and $\boldsymbol{\eta}_L$ is the QS prediction. The training data of the lift coefficient is stored in the vector $\mathbf{C}_L$.

As shown in Eq. (3), the training data $(\boldsymbol{\alpha}, \mathbf{C}_L)$ is required to train the model. This data must replicate the conditions under which the model will be used. Since the current study focuses on buffeting analysis, we devise a bespoke procedure for forced vibration under free-stream turbulence in a CFD setting. The motion angles α_h and α_a are generated based on the inverse Fourier Transform of a spectrum defined in Fig. 3 (left). Specifically, we set minimum and maximum reduced velocities ($V_{r,min}$ and $V_{r,max}$) for linearly varying amplitude, with a maximum of 1. At low reduced velocities (high frequency), we attenuate the spectrum because high-frequency motion cannot have high amplitude due to numerical issues. The randomly generated signal, based on the prescribed spectrum, is then scaled through its variance to achieve a prescribed motion amplitude. For free-stream turbulence, we use the standard von Kármán spectrum to generate random signals for the upstream velocity. To accommodate various turbulence intensities, we use two separate signals for two separate CFD simulations and combine the training data, as seen in Fig. 3 (right).

Our training data ultimately consists of lift coefficients for several sets of input motion and gust angles, each with various motion amplitudes and turbulence intensities.

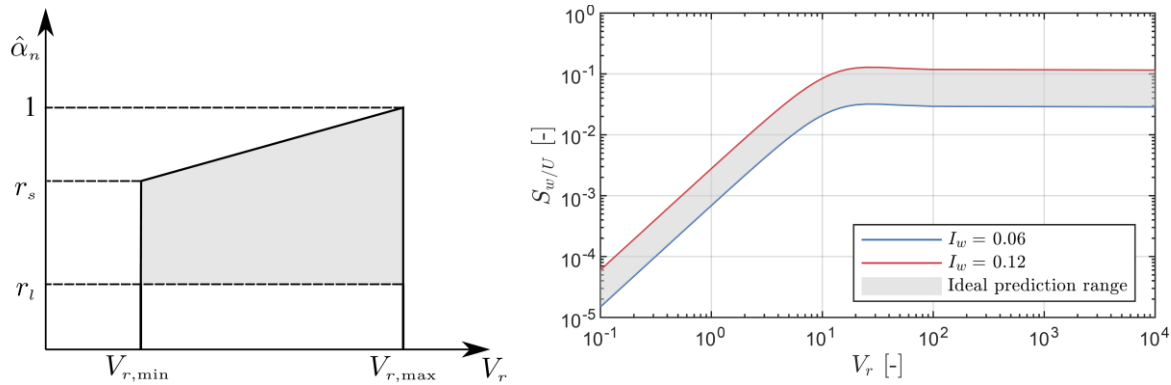


Figure 3. Training: Input motion- and gust-induced angles (top) and lift and moment coefficients (bottom).

Once trained, the hybrid model can be used for prediction of the lift coefficient at prediction points $\boldsymbol{\alpha}_i^*$ as:

$$p(\mathbf{f}_L^* | \boldsymbol{\alpha}^*, \mathbf{C}_L, \boldsymbol{\alpha}, \boldsymbol{\eta}_L; \theta) \sim N(\boldsymbol{\eta}_L^* + \mathbf{K}_{LL}^T (\mathbf{K}_L + \sigma_L^2 \mathbf{I})^{-1} (\mathbf{C}_L - \boldsymbol{\eta}_L), \mathbf{K}_L^* - \mathbf{K}_{LL}^T (\mathbf{K}_L + \sigma_L^2 \mathbf{I})^{-1} \mathbf{K}_{LL}^*), \quad (4)$$

where $\mathbf{K}_{LL}^*$ and $\mathbf{K}_L^*$ are constructed based on a kernel function k , and $\boldsymbol{\eta}_L^*$ is the quasi-steady prediction. The formulation of the moment is similar.

3. APPLICATION TO BRIDGE AERODYNAMICS

The presented hybrid model is applied to predict the aerodynamic forces and buffeting response of the Great Belt Bridge deck, utilizing CFD data from Kavrakov and Morgenthal (2018). Figure 4 illustrates a particle map of the CFD analyses of the section under free-stream turbulence.

The GP model is trained based on forced vibration concurrently under free-stream turbulence, using two signals with different amplitudes stacked together. The motion-induced angles have standard deviations of $\sigma_{\alpha_h} = \sigma_{\alpha_a} = 2.5$ and 15 deg, with a reduced velocity range of $2 \leq V_r \leq 16$ ($V_r = U/fB$; f =frequency), while the gust-induced angles have 6% and 12% isotropic turbulence following von Kármán spectra. Figure 5 illustrates a sample of the gust- and motion-induced signals, along with the corresponding training signals of the lift and moment coefficients.

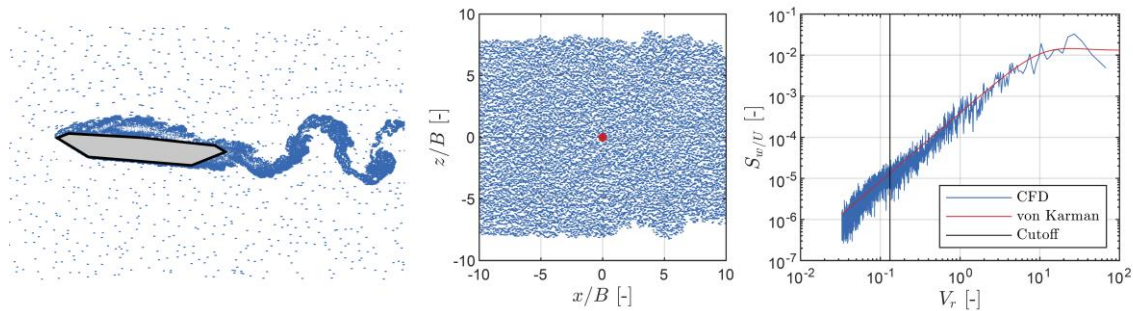


Figure 4. Particle map of a section subjected to turbulent free-stream (left); particle map of the free-stream velocity without a section (centre) – the red point is tracking location of the wind velocity; spectrum of wind velocity (right).

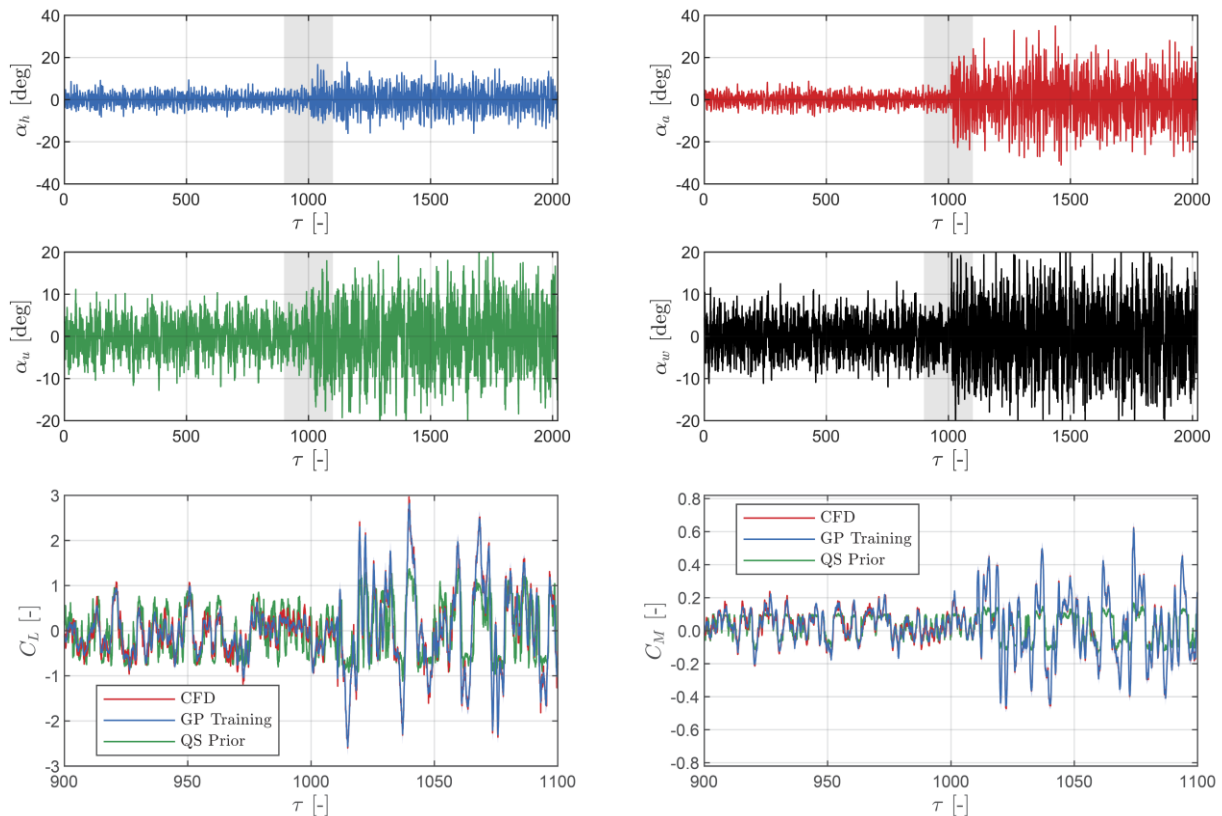


Figure 5. Training: Input motion- and gust-induced angles (top/centre) and lift and moment coefficients (bottom).

The model is used to predict motion-induced and gust-induced forces during forced excitation. The static wind coefficients provide a useful method to assess the model's robustness for reduced velocities outside the training range (i.e., at infinity). By imposing a constant rotation, the lift and moment coefficients are obtained and depicted in Fig. 6. Interestingly, there is a difference despite having the QS model as an aerodynamic prior— theoretically, the data-driven part should be zero. However, there is a small contribution from the data-driven part in the lift and moment, but it remains within the predictive confidence interval.

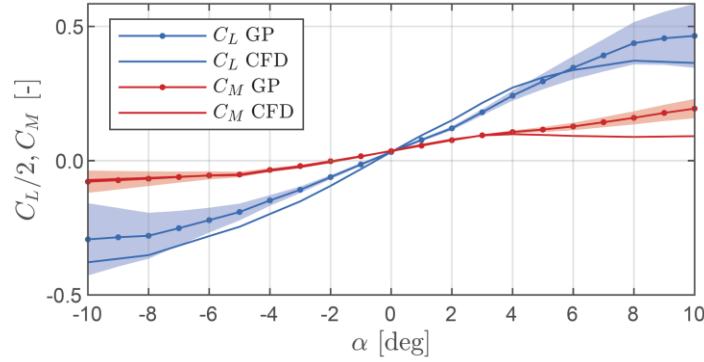


Figure 6. Prediction: Static aerodynamic coefficients. The shaded area represents the 99% confidence interval for the GP model.

The linear frequency dependence, or fluid memory, of both gust- and motion-induced forces is of primary importance in bridge aeroelasticity. Using aerodynamic admittance (χ_L - lift; χ_M - moment) for gust-induced forces and aerodynamic derivatives (H_i^* , A_i^*) for motion-induced forces, we examine how well the hybrid model captures linear fluid memory. Figure 7 shows the aerodynamic admittance, obtained from a random free-stream turbulence of 10%, which differs from the training set values of 6% and 10%. A good correspondence is observed between the CFD and GP models. Notably, vortex shedding appears in the CFD model within the range $0.3 \leq V_r \leq 1$. This contribution is filtered out by the GP prediction, as the model is designed to exclude vortex shedding forces.

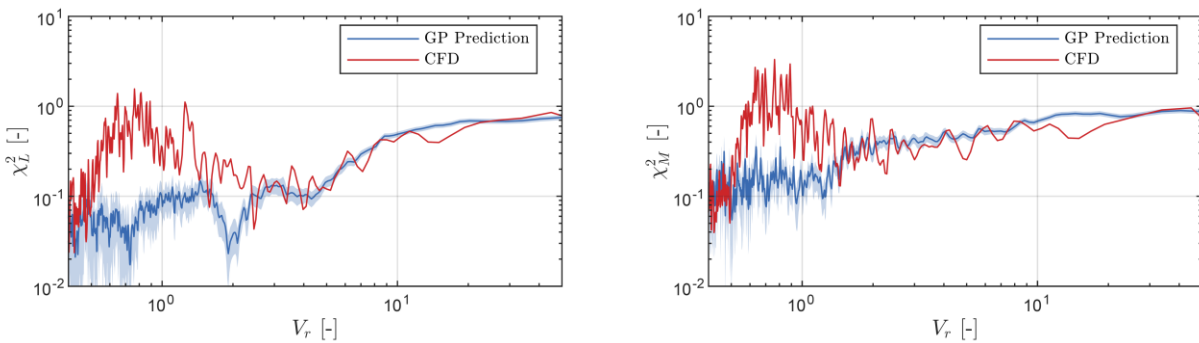


Figure 7. Prediction: Aerodynamic admittance. The shaded area represents the 99% confidence interval for the GP model.

The aerodynamic derivatives were obtained by forcing the section to oscillate sinusoidally with an amplitude of 1 degree, which is sufficiently small to cover the linear part of the aerodynamics. Figure 8 depicts the aerodynamic derivatives and the interval corresponding to 1000 samples for

the GP model. A good correspondence is observed between the CFD and the hybrid GP model. This is an important property, as it demonstrates that the GP model can isolate the motion-induced part of the forces, despite being trained on coupled input.

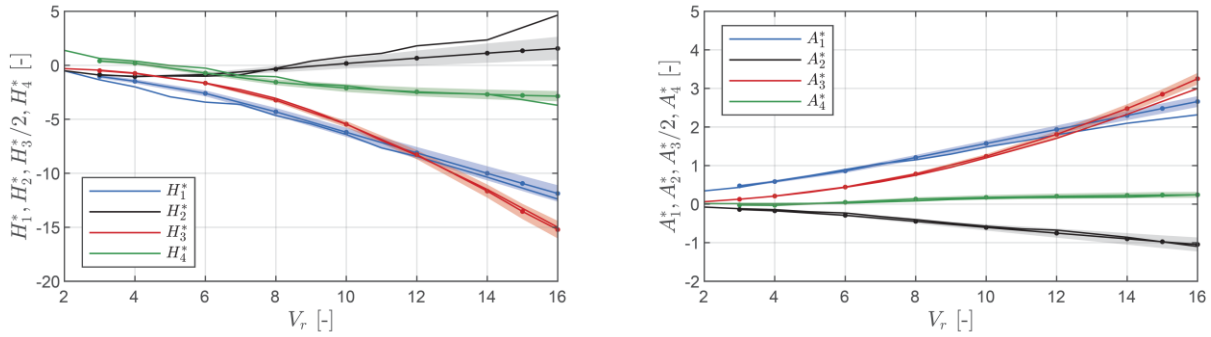


Figure 8. Prediction: Aerodynamic derivatives. The shaded area represents the 99% confidence interval for the GP model.

Nonlinear fluid memory is typically difficult to capture with semi-analytical models due to the nonlinear dependence of self-excited forces on both frequency and amplitude. We demonstrate the model's capability by forcing the section to sinusoidally rotate with a large amplitude of 10 degrees at a reduced velocity of $V_r=10$. Figure 9 depicts the moment time history (left) and the corresponding Fourier transform (right) for the CFD and GP models, along with the QS prior. It can be seen that the CFD and GP models yield similar results, with the GP model successfully capturing the higher-order harmonics—a typical manifestation of nonlinear fluid memory. Interestingly, the QS model significantly underestimates the force at positive amplitudes, indicating that the model discrepancy is accurately captured by the data-driven part of the hybrid GP model.

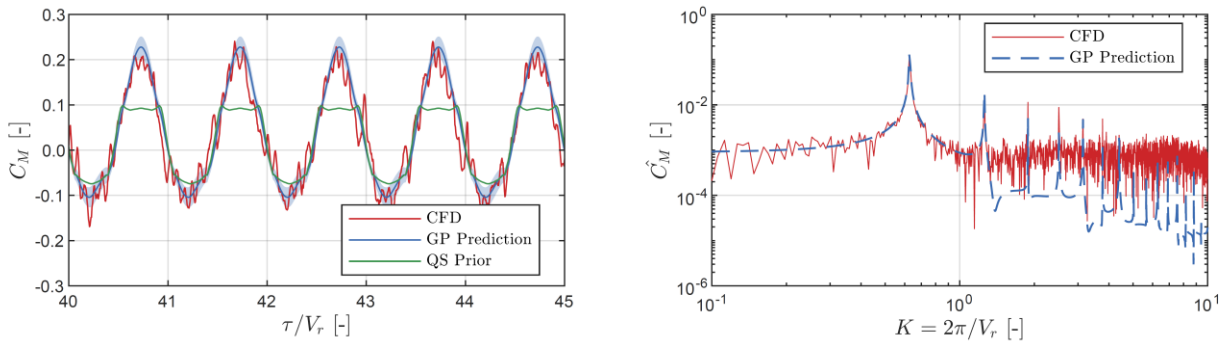


Figure 9. Prediction: Motion-induced moment time history (right) and its corresponding Fourier spectrum (left) for forced sinusoidal rotation with amplitude of $\alpha=10$ deg, at reduced velocity of $V_r=10$.

Finally, buffeting analysis is performed by subjecting the GP model to the tracked wind velocities from a CFD simulation without a cross section (see Fig. 4). The input wind histories have mean wind speeds between 20 m/s and 50 m/s, each lasting 600 seconds, with a turbulence intensity of 10%. It's worth noting that the turbulence intensity differs from the ones of 6% and 12% used during training.

Figure 10 displays a sample time histories of the vertical displacements and their corresponding Welch spectrum at velocities of 30 and 60 m/s, comparing results from the CFD and the GP models. Figure 11 illustrates the standard deviation of the response over the selected wind speed range between 20 and 60 m/s.

Good agreement can be observed for both rotational and vertical displacements. The frequency content is accurately captured, including the coupling term at the rotational frequency in the vertical displacements at 60 m/s. However, there are slightly larger discrepancies noted for the rotation at 60 m/s, which is also evident in the standard deviation. The exact reason for this is difficult to pinpoint—while large amplitudes are involved, it is conjectured that non-stationary effects, which are challenging for the hybrid GP model to capture, may be responsible. It should also be noted that the input velocity for the hybrid GP model is taken from the centre of the domain in a simulation without a section. At such large amplitudes, it is expected that the section in the CFD simulation experiences varying wind velocities in the fluid domain.

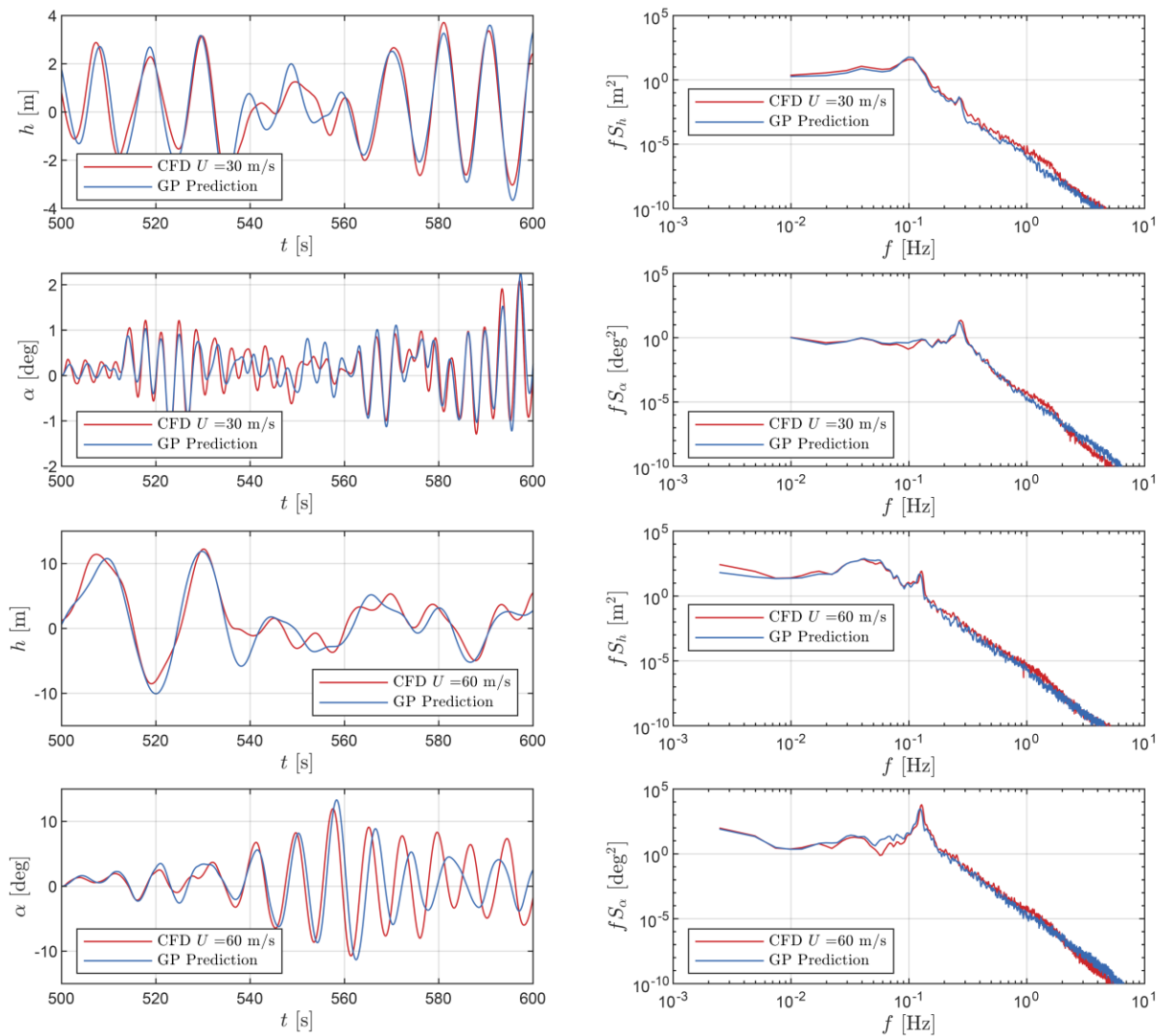


Figure 10. Prediction: Sample of time histories (left) and power spectra (right) of the vertical displacements during buffeting response at $U = 30$ m/s and 60 m/s and turbulence intensity of 10 %.

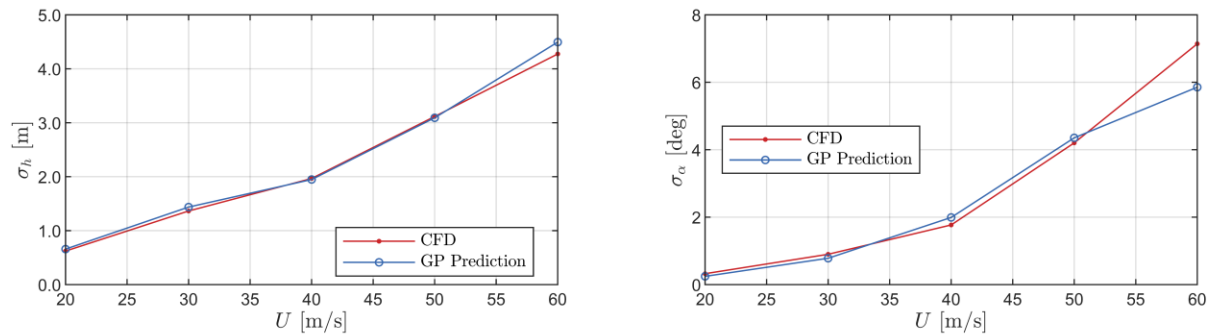


Figure 11. Prediction: Standard deviation of the vertical displacements (left) and rotation (right) of the buffeting response at various wind speeds with turbulence intensity of 10 %.

4. CONCLUSIONS

The study presents a hybrid GP model of the aerodynamic forces, consisting of a semi-analytical and a data-driven part. The semi-analytical part was covered by the standard nonlinear QS semi-analytical model, while GP regression was utilised to cover the model discrepancy learned through data. A bespoke procedure for generating training signals was devised using motion and free-stream turbulence concurrently. Both forced vibration and buffeting analyses were performed to study the capabilities of presented methodology.

The model's predictions were verified for both linear and nonlinear aerodynamics. It was demonstrated that the model robustly captures the static aerodynamic coefficients, effectively predicting even outside the training range at infinite reduced velocity. Verification of linear unsteady aerodynamics was achieved through aerodynamic admittance and aerodynamic derivatives. Furthermore, the model successfully captured higher-order harmonics in the self-excited forces. With these individual effects verified, the buffeting response showed good agreement between the CFD simulations and the presented GP model.

The presented methodology can be extended to include drag forces and lateral motion. By utilizing the strip assumption, it can be directly applied to pseudo-3D buffeting analyses of line-like structures, considering multiple modes.

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